# Ramsey Quantifiers in Linear Arithmetics

Pascal Bergsträßer[A]

[A]University of Kaiserslautern-Landau, Kaiserslautern, Germany
bergstraesser@cs.uni-kl.de

We study Satisfiability Modulo Theories (SMT) enriched with the so-called Ramsey quantifiers, which assert the existence of cliques (complete graphs) in the graph induced by some formulas. The extended framework is known to have applications in proving program termination (in particular, whether a transitive binary predicate is well-founded), and monadic decomposability of SMT formulas. Our main result is a new algorithm for eliminating Ramsey quantifiers from three common SMT theories: Linear Integer Arithmetic (LIA), Linear Real Arithmetic (LRA), and Linear Integer Real Arithmetic (LIRA). In particular, if we work only with existentially quantified formulas, then our algorithm runs in polynomial-time and produces a formula of linear size. One immediate consequence is that checking well-foundedness of a given formula in the aforementioned theory defining a transitive predicate can be straightforwardly handled by highly optimized SMT-solvers. We show also how this provides a uniform semi-algorithm for verifying termination and liveness with completeness guarantee (in fact, with an optimal computational complexity) for several well-known classes of infinite-state systems, which include succinct timed systems, one-counter systems, and monotonic counter systems. Another immediate consequence is a solution to an open problem on checking monadic decomposability of a given relation in quantifier-free fragments of LRA and LIRA, which is an important problem in automated reasoning and constraint databases. Our result immediately implies decidability of this problem with an optimal complexity (coNP-complete) and enables exploitation of SMT-solvers. It also provides a termination guarantee for the generic monadic decomposition algorithm of Veanes et al. for LIA, LRA, and LIRA. We report encouraging experimental results on a prototype implementation of our algorithms on micro-benchmarks.

This is joint work with Moses Ganardi, Anthony W. Lin, and Georg Zetzsche.