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Remarks on Parikh-recognizable omega-languages (Extended Abstract)

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Abstract

Several variants of Parikh automata on infinite words were recently introduced by Guha et al. [FSTTCS, 2022]. We show that one of these variants coincides with blind counter machine as introduced by Fernau and Stiebe [Fundamenta Informaticae, 2008]. Fernau and Stiebe showed that every ω-language recognized by a blind counter machine is of the form $\bigcup_i U_i V_i^\omega$ for Parikh recognizable languages U_i, V_i, but blind counter machines fall short of characterizing this class of ω-languages. They posed as an open problem to find a suitable automata-based characterization. We introduce several additional variants of Parikh automata on infinite words that yield automata characterizations of classes of ω-language of the form $\bigcup_i U_i V_i^\omega$ for all combinations of languages U_i, V_i being regular or Parikh-recognizable. When both U_i and V_i are regular, this coincides with Büchi's classical theorem. We study the effect of ε-transitions in all variants of Parikh automata and show that almost all of them admit ε-elimination. Finally we study the classical decision problems with applications to model checking.

1. Introduction

Finite automata find numerous applications in formal language theory, logic, verification, and many more, in particular due to their good closure properties and algorithmic properties. To enrich this spectrum of applications even more, it has been a fruitful direction to add features to finite automata to capture also situations beyond the regular realm.

One such possible extension of finite automata with counting mechanisms has been introduced by Greibach in her study of blind and partially blind (one-way) multicounter machines [13]. Blind multicounter machines are generalized by weighted automata as introduced in [20]. Parikh automata (PA) were introduced by Klaedtke and Rueß in [19]. A PA is a non-deterministic finite automaton that is additionally equipped with a semi-linear set C, and every transition is equipped with a d-tuple of non-negative integers. Whenever an input word is read, d counters are initialized with the values 0 and every time a transition is used, the counters are incremented by the values in the tuple of the transition accordingly. An input word is accepted if the PA ends in an accepting

The full version of this paper can be found on arXiv [14] state and additionally, the resulting d-tuple of counter values lies in C. We call such a pair an accepting configuration. Klaedtke and Rueß showed that PA are equivalent to weighted automata over the group $(\mathbb{Z}^k, +, \mathbf{0})$, and hence equivalent to Greibach's blind multicounter machines, as well as to reversal bounded multicounter machines [1, 17]. Recently it was shown that these models can be translated into each other using only logarithmic space [2]. In this work we call the class of languages recognized by any of these models *Parikh recognizable*. Klaedtke and Rueß [19] showed that the class of Parikh recognizable languages is precisely the class of languages definable in weak existential monadic second-order logic of one successor extended with linear cardinality constraints. On finite words, blind counter automata, Parikh automata and related models have been investigated extensively, extending [13, 19] for example by affine PA and PA on letters [4, 5], bounded PA [6], two-way PA [12], PA with a pushdown stack [18] as well as a combination of both [7], history-deterministic PA [8], automata and grammars with valences [9, 16], and several algorithmic applications, e.g. in the context of path logics for querying graphs [11].

Guha et al. [15] introduced *safety, reachability, Büchi- and co-Büchi Parikh automata*. These models provide natural generalization of studied automata models with Parikh conditions on infinite words. One shortcoming of safety, reachability and co-Büchi Parikh automata is that they do not generalize Büchi automata, that is, they cannot recognize all ω-regular languages. The non-emptiness problem, which is highly relevant for model checking applications, is undecidable for safety and co-Büchi Parikh automata. Furthermore, none of these models has ω-closure, meaning that for every model there is a Parikh-recognizable language (on finite words) L such that L^ω is not recognizable by any of these models. They raised the question whether (appropriate variants of) Parikh automata on infinite words have the same expressive power as blind counter automata on infinite words.

Büchi's famous theorem states that ω-regular languages are characterized as languages of the form $\bigcup_i U_i V_i^\omega$, where the U_i and V_i are regular languages [3]. As a consequence of the theorem, many properties of ω-regular languages are inherited from regular languages. For example, the non-emptiness problem for Büchi automata can basically be solved by testing non-emptiness for nondeterministic finite automata. In their systematic study of blind counter automata, Fernau and Stiebe [10] considered the class $\mathcal{K}_*$, the class of ω-languages of the form $\bigcup_i U_i V_i^\omega$ for Parikh-recognizable languages U_i and V_i. They proved that the class of ω-languages recognizable by blind counter machines is a proper subset of the class $\mathcal{K}_*$. They posed as an open problem to provide automata models that capture classes of ω-languages of the form $\bigcup_i U_i V_i^\omega$ where U_i and V_i are described by a certain mechanism.

2. Results

In this work, we propose *reachability-regular Parikh automata*, *limit Parikh automata*, and *reset Parikh automata* as new automata models.

We pick up the question of Fernau and Stiebe [10] to consider classes of ω-languages of the form $\bigcup_i U_i V_i^\omega$ where U_i and V_i are described by a certain mechanism. We define the four classes $\mathcal{L}^\omega_{\mathsf{Reg},\mathsf{Reg}}$, $\mathcal{L}^\omega_{\mathsf{PA},\mathsf{Reg}}$, $\mathcal{L}^\omega_{\mathsf{Reg},\mathsf{PA}}$ and $\mathcal{L}^\omega_{\mathsf{PA},\mathsf{PA}}$ of ω-languages of the form $\bigcup_i U_i V_i^\omega$, where the U_i, V_i are regular or Parikh-recognizable languages of finite words, respectively. By Büchi's theorem the class $\mathcal{L}^\omega_{\mathsf{Reg},\mathsf{Reg}}$ is the class of ω-regular languages.

Guha et al. [15] showed that the class of Büchi PA-recognizable ω-languages is a strict subclass of $\mathcal{L}^{\omega}_{\mathsf{PA},\mathsf{PA}}$. First we show the following characterization.

Theorem 2.1 *The following are equivalent for all ω-languages $L \subseteq \Sigma^{\omega}$:*

1. *L is Büchi PA-recognizable.*

2. *L is of the form $\bigcup_i U_i V_i^{\omega}$, where $U_i \in \Sigma^*$ is Parikh-recognizable and $V_i \in \Sigma^*$ is recognized by a PA where the initial state is the only accepting state and C is a linear set without base vector.*

We next show that the newly introduced reachability-regular Parikh automata, which are a small modification of reachability Parikh automata (as introduced by Guha et al. [15]) capture exactly the class $\mathcal{L}^{\omega}_{\mathsf{PA},\mathsf{Reg}}$. Such an automaton accepts an infinite word if it has a prefix that leads to an accepting configuration, and an accepting state is seen infinitely often. This model turns out to be equivalent to limit Parikh automata. Such an automaton utilizes semi-linear sets over $\mathbb{N}^d \cup \{\infty\}$ and computes the Parikh image over the whole infinite word component-wise. This model was hinted at in the concluding remarks of [19].

Theorem 2.2 *The following are equivalent for all ω-languages $L \subseteq \Sigma^{\omega}$.*

1. *L is of the form $\bigcup_i U_i V_i^{\omega}$, where $U_i \in \Sigma^*$ is Parikh-recognizable, and $V_i \subseteq \Sigma^*$ is regular.*

2. *L is limit PA-recognizable.*

3. *L is reachability-regular.*

Fully resolving the classification of the above mentioned classes we introduce reset Parikh automata. Such an automaton resets the counters every time an accepting state is seen and the current counter values lie in the semi-linear set, and accepts an infinite word if it resets infinitely often. In contrast to all other Parikh models, these are closed under the ω-operation, while maintaining all algorithmic properties of PA (in particular, non-emptiness is NP-complete and hence decidable). We show that the class of Reset-recognizable ω-languages is a strict superclass of $\mathcal{L}^{\omega}_{\mathsf{PA},\mathsf{PA}}$. We show that appropriate graph-theoretic restrictions of reset Parikh automata exactly capture the classes $\mathcal{L}^{\omega}_{\mathsf{PA},\mathsf{PA}}$ and $\mathcal{L}^{\omega}_{\mathsf{Reg},\mathsf{PA}}$, yielding the first automata characterizations for these.

Theorem 2.3 *The following are equivalent for all ω-languages $L \subseteq \Sigma^{\omega}$.*

1. *L is of the form $\bigcup_i U_i V_i^{\omega}$, where $U_i, V_i \subseteq \Sigma^*$ are Parikh-recognizable.*

2. *L is recognized by a strong reset PA $\mathcal{A}$ with the property that accepting states appear only in the leaves of the condensation of $\mathcal{A}$, and there is at most one accepting state per leaf.*

Theorem 2.4 *The following are equivalent for all ω-languages $L \subseteq \Sigma^{\omega}$.*

1. *L is of the form $\bigcup_i U_i V_i^{\omega}$, where $U_i \subseteq \Sigma^*$ is regular and $V_i \subseteq \Sigma^*$ is Parikh-recognizable.*

2. *L is recognized by a strong reset PA $\mathcal{A}$ with the following properties.*

(a) *At most one state q per leaf of the condensation of $\mathcal{A}$ may have incoming transitions from outside the leaf, this state q is the only accepting state in the leaf, and there are no accepting states in non-leaves.*

(b) *only transitions connecting states in a leaf may be labeled with a non-zero vector.*

The automata models introduced by Guha et al. [15] do not have ε-transitions, while blind counter machines have such transitions. Towards answering the question of Guha et al. we study the effect of ε-transitions in all Parikh automata models. We show that all models except safety and co-Büchi Parikh automata admit ε-elimination.

Theorem 2.5 *ε-reachability, ε-reachability-regular, ε-limit PA, Büchi PA and reset PA admit ε-elimination.*

This in particular answers the question of Guha et al. [15] whether blind counter machines and Büchi Parikh automata have the same expressive power over infinite words affirmative, as we can easily show that blind counter machines and ε-Büchi PA are equivalent.

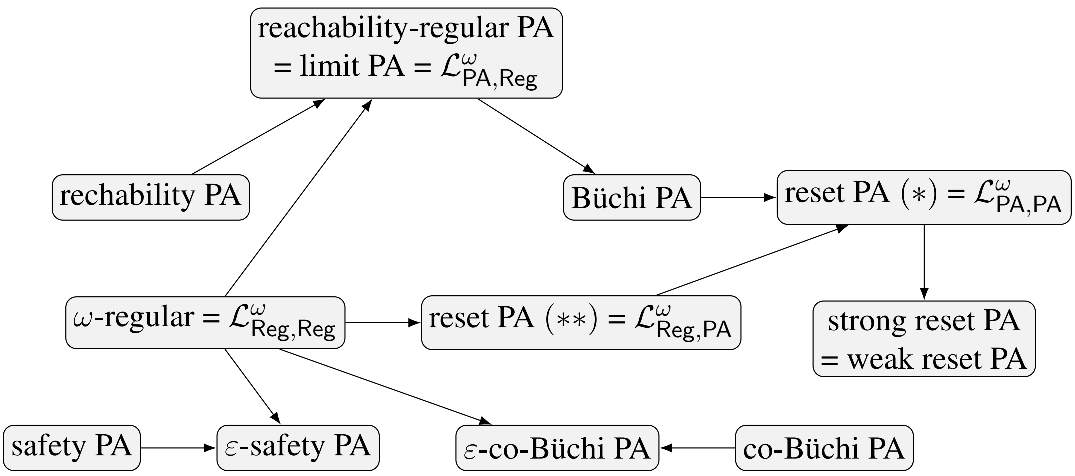
Lemma 2.6 *Blind counter machines and Büchi PA are equivalent.*

We show that safety and co-Büchi automata with ε-transitions are strictly more powerful than their variants without ε-transitions, and in particular, they give the models enough power to recognize all ω-regular languages.

Lemma 2.7 *Every ω-regular language is ε-safety PA and ε-co-Büchi PA recognizable.*

Corollary 2.8 *ε-safety PA and ε-co-Büchi PA do not admit ε-elimination.*

Find an overview of these results in Figure 1.



$(*)$ At most one state q per leaf of $C(\mathcal{A})$ may have incoming transitions from outside the leaf, this state q is the only accepting state in the leaf, and there are no accepting states in non-leaves;
$(**)$ and only transitions connecting states in leaves may be labeled with non-zero vectors.

Figure 1: Overview of our results. Arrows mean strict inclusions.

References

[1] B. S. BAKER, R. V. BOOK, Reversal-bounded multipushdown machines. *Journal of Computer and System Sciences* **8** (1974) 3, 315–332.

[2] P. BAUMANN, F. D'ALESSANDRO, M. GANARDI, O. IBARRA, I. MCQUILLAN, L. SCHÜTZE, G. ZETZSCHE, Unboundedness Problems for Machines with Reversal-Bounded Counters. In: *Foundations of Software Science and Computation Structures*. Springer Nature Switzerland, Cham, 2023, 240–264.

[3] J. R. BÜCHI, Weak Second-Order Arithmetic and Finite Automata. *Mathematical Logic Quarterly* **6** (1960) 1-6, 66–92.

[4] M. CADILHAC, A. FINKEL, P. MCKENZIE, On the Expressiveness of Parikh Automata and Related Models. In: *Third Workshop on Non-Classical Models for Automata and Applications - NCMA 2011*. books@ocg.at 282, Austrian Computer Society, 2011, 103–119.

[5] M. CADILHAC, A. FINKEL, P. MCKENZIE, Affine Parikh automata. *RAIRO Theor. Informatics Appl.* **46** (2012) 4, 511–545.

[6] M. CADILHAC, A. FINKEL, P. MCKENZIE, Bounded Parikh Automata. *Int. J. Found. Comput. Sci.* **23** (2012) 8, 1691–1710.

[7] L. DARTOIS, E. FILIOT, J. TALBOT, Two-Way Parikh Automata with a Visibly Pushdown Stack. In: *Foundations of Software Science and Computation Structures - 22nd International Conference, FOSSACS 2019*. Lecture Notes in Computer Science 11425, Springer, 2019, 189–206.

[8] E. ERLICH, S. GUHA, I. JECKER, K. LEHTINEN, M. ZIMMERMANN, History-deterministic Parikh Automata. *arXiv preprint arXiv:2209.07745* (2022).

[9] H. FERNAU, R. STIEBE, Sequential grammars and automata with valences. *Theoretical Computer Science* **276** (2002) 1, 377–405.

[10] H. FERNAU, R. STIEBE, Blind Counter Automata on omega-Words. *Fundam. Inform.* **83** (2008), 51–64.

[11] D. FIGUEIRA, L. LIBKIN, Path Logics for Querying Graphs: Combining Expressiveness and Efficiency. In: *Proceedings of the 2015 30th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*. LICS '15, IEEE, 2015, 329–340.

[12] E. FILIOT, S. GUHA, N. MAZZOCCHI, Two-Way Parikh Automata. In: *39th IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science, FSTTCS 2019*. LIPIcs 150, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2019, 40:1–40:14.

[13] S. A. GREIBACH, Remarks on blind and partially blind one-way multicounter machines. *Theoretical Computer Science* **7** (1978) 3, 311–324.

[14] M. GROBLER, L. SABELLEK, S. SIEBERTZ, Remarks on Parikh-recognizable omega-languages. *arXiv preprint arXiv:2307.07238* (2023).

[15] S. GUHA, I. JECKER, K. LEHTINEN, M. ZIMMERMANN, Parikh Automata over Infinite Words. In: *42nd IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS 2022)*. Leibniz International Proceedings in Informatics (LIPIcs) 250, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2022, 40:1–40:20.

[16] H. J. HOOGEBOOM, Context-Free Valence Grammars - Revisited. In: *Developments in Language Theory*. Springer Berlin Heidelberg, Berlin, Heidelberg, 2002, 293–303.

[17] O. H. IBARRA, Reversal-Bounded Multicounter Machines and Their Decision Problems. *J. ACM* **25** (1978) 1, 116—133.

[18] W. KARIANTO, Parikh automata with pushdown stack. *Diplomarbeit, RWTH Aachen* (2004).

[19] F. KLAEDTKE, H. RUESS, Monadic Second-Order Logics with Cardinalities. In: *Automata, Languages and Programming*. Springer, Berlin, Heidelberg, 2003, 681–696.

[20] V. MITRANA, R. STIEBE, Extended finite automata over groups. *Discrete Applied Mathematics* **108** (2001) 3, 287–300.