

# $k$ -Universality of Regular Languages

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## Abstract

A subsequence of a word  $w$  is a word  $u$  such that  $u = w[i_1]w[i_2] \dots w[i_k]$ , for some set of indices  $1 \leq i_1 < i_2 < \dots < i_k \leq |w|$ . A word  $w$  is  $k$ -subsequence universal over an alphabet  $\Sigma$  if every word in  $\Sigma^k$  appears in  $w$  as a subsequence. In this talk, we focus on the intersection between the set of  $k$ -subsequence universal words over some alphabet  $\Sigma$  and regular languages over  $\Sigma$ . We call a regular language  $L$   $k$ - $\exists$ -subsequence universal if there exists a  $k$ -subsequence universal word in  $L$ , and  $k$ - $\forall$ -subsequence universal if every word of  $L$  is  $k$ -subsequence universal. We present algorithms solving the problems of deciding if a given regular language, represented by a finite automaton accepting it, is  $k$ - $\exists$ -subsequence universal and, respectively, if it is  $k$ - $\forall$ -subsequence universal, for a given number  $k$ . The algorithms are FPT w.r.t. the size of the input alphabet, and their run-time does not depend on  $k$ ; they run in polynomial time in the number  $n$  of states of the input automaton when the size of the input alphabet is  $O(\log n)$ . Moreover, we show that the problem of deciding if a given regular language is  $k$ - $\exists$ -subsequence universal is NP-complete, when the language is over a large alphabet. Further, we provide algorithms for counting the number of  $k$ -subsequence universal words (paths) accepted by a given deterministic (respectively, nondeterministic) finite automaton, and ranking an input word (path) within the set of  $k$ -subsequence universal words accepted by a given finite automaton.

The paper on which this talk is based is accepted at ISAAC 2023 [1].

## References

- [1] D. ADAMSON, P. FLEISCHMANN, A. HUCH, T. KOSS, F. MANEA, D. NOWOTKA,  $k$ -Universality of Regular Languages. In: *ISAAC 2023, Proceedings*. LIPIcs (and full version on Arxiv), to appear, 2023.