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Rational trace relations

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Rational relations form a classical and well-studied concept (cf. [15, 14, 4, 17]) that embraces homomorphisms, inverse homomorphisms as well as substitutions. Rational relations appear in the study of automatic structures [1, 18, 27], rational Kripke frames [3], graph databases [2], the representation of infinite graphs and automata [19, 25, 28, 26, 8, 7], and natural language processing [20]. One particular application of rational relations can be found in the theory of pushdown systems: the reachability relation is prefix recognizable [10, 16] and therefore a rational relation which implies that forwards and backwards reachability preserve the regularity of a set of configurations ([6] provides an alternative proof for the backwards reachability).

Also the second theme of this paper has a long and diverse research history starting with Cartier and Foata's work in combinatorics [9] and Mazurkiewicz's ideas about the semantics of concurrent systems [24] that he modelled as equivalence classes of words, called traces today. Much of the work in computer science has concentrated on recognizable sets of traces, on model checking and synthesis problems, and on combinatorics, see [11] for a comprehensive presentation of the theory of traces; many of these results have been extended to more general concurrent systems like concurrent automata (cf., e.g. [13]), message passing automata [23], and other abstract models of distributed automata (e.g. [12, 5]).

Recently, Köcher and the current author considered a generalization of pushdown systems where the stack's contents is not a word, but a trace [22]; these systems were called *cooperating pushdown systems* or cPDS. Our main results state that the forwards reachability relation preserves the rationality and the backwards reachability the recognizability of sets of configurations (but not vice versa). While the reachability relation of a classical pushdown system is prefix recognizable, we also observed that this is not the case for cPDS. In addition, Köcher [21] infered from the main result that the reachability relation is a rational trace relation, but it was not clear whether this rationality could be used to prove the preservation results as in the word case.

The first insights of this work show that rational trace relations differ significantly from rational word relations since they do not preserve rationality nor recognizability nor do they compose. To overcome these deficits, we study the restricted class $lc\mathcal{R}$ of *left-closed rational trace relations* and demonstrate that these relations enjoy many of the important properties of rational word relations: they preserve rationality (but not recognizability), their inverses preserve recognizability (but not rationality), they compose, and any rational relation is the composition of the inverse of a relation from $lc\mathcal{R}$ and a relation from $lc\mathcal{R}$.

From lemmas in [22], it follows that the reachability relation of a cPDS is a finite union of compositions of certain trace relations that resemble prefix-recognizable word relations.

We show that these "building blocks" are left-closed rational. It follows that the reachability relation of a cPDS is left-closed rational. Hence forwards reachability preserves rationality and backwards reachability preserves recognizability or sets of configurations (the main results from [22]).

Thus, the talk introduces a new class of relations and uses them to prove (parts of) the results from [22] in a more uniform and (as the author hopes) transparent way.

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