Reinforcement Learning with Reward Machines

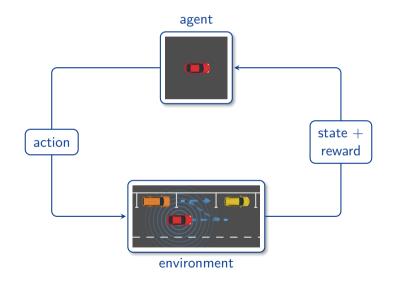
Daniel Neider





Theorietag "Automaten und Formale Sprachen" RPTU/MPI-SWS, Kaiserslautern, Germany 4 October 2023

Reinforcement Learning

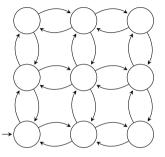




Actions $A = \{\uparrow, \downarrow, \leftarrow, \rightarrow\}$ Labels $\mathcal{P} = \{\diamondsuit, \boxdot\}$

~

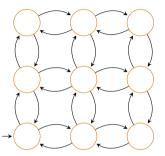




Actions A = {↑, ↓, ←, →}
Labels $\mathcal{P} = {♀, ■}$

 $\mathcal{M} = (S, s_I, A, \mathcal{P}, p, L, R, \gamma)$



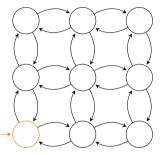


Actions A = {↑, ↓, ←, →}
Labels $\mathcal{P} = {♀, ■}$

 $\mathcal{M} = (S, s_I, A, \mathcal{P}, p, L, R, \gamma)$

set S of states



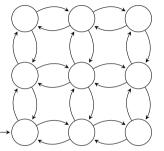


Actions A = {↑, ↓, ←, →}
 Labels P = {♀, □>}

 $\mathcal{M} = (S, \boldsymbol{s_l}, \boldsymbol{A}, \mathcal{P}, \boldsymbol{p}, \boldsymbol{L}, \boldsymbol{R}, \boldsymbol{\gamma})$

initial state $s_l \in S$



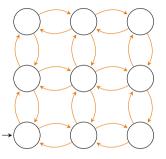


Actions A = {↑, ↓, ←, →}
Labels $\mathcal{P} = {♀, ■}$

 $\mathcal{M} = (S, s_I, \mathcal{A}, \mathcal{P}, p, L, R, \gamma)$

actions A and labels \mathcal{P}





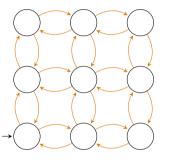
- Actions $A = \{\uparrow, \downarrow, \leftarrow, \rightarrow\}$
- Labels $\mathcal{P} = \{\mathbf{Q}, \mathbf{m}\}$

 $\mathcal{M} = (S, s_I, A, \mathcal{P}, \frac{p}{P}, L, R, \gamma)$

transition function $p \colon S \times A \times S \rightarrow [0, 1]$







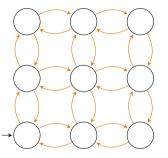
- Actions $A = \{\uparrow, \downarrow, \leftarrow, \rightarrow\}$
- Labels $\mathcal{P} = \{\mathbf{Q}, \mathbf{m}\}$

 $\mathcal{M} = (S, s_I, A, \mathcal{P}, p, \underline{L}, R, \gamma)$

labeling function $L: S \times A \times S \rightarrow \mathcal{P}$







Actions
$$A = \{\uparrow, \downarrow, \leftarrow, \rightarrow\}$$

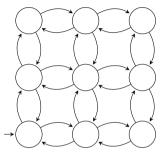
• Labels $\mathcal{P} = \{\mathbf{Q}, \mathbf{m}\}$

$$\mathcal{M} = (S, s_I, A, \mathcal{P}, p, L, \mathbb{R}, \gamma)$$

reward function $R: S \times A \times S \rightarrow \mathbb{R}$







Actions A = {↑, ↓, ←, →}
Labels $\mathcal{P} = {♀, ■}$

 $\mathcal{M} = (S, s_I, A, \mathcal{P}, p, L, R, \gamma)$

discount factor $\gamma \in (0, 1)$

A Very, Very Brief Introduction to Q-Learning

Find a (probabilistic) policy $\pi: S \times A \rightarrow [0,1]$ maximizing the expected discounted reward

$$\mathbb{E}_{\pi} \Big[\sum_{i=0}^{k} \gamma^{i} \cdot R(s_{i}, a_{i+1}, s_{i+1}) \Big]$$

of every trajectory $s_0a_0s_1\ldots s_{k+1}$, $k\in\mathbb{N}$, through the MDP

A Very, Very Brief Introduction to Q-Learning

Find a (probabilistic) policy $\pi \colon S \times A \to [0,1]$ maximizing the expected discounted reward

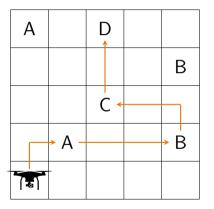
$$\mathbb{E}_{\pi}\left[\sum_{i=0}^{k} \gamma^{i} \cdot R(s_{i}, a_{i+1}, s_{i+1})\right]$$

of every trajectory $s_0a_0s_1\ldots s_{k+1}$, $k\in\mathbb{N}$, through the MDP

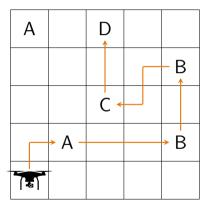
Q-Learning

- 1. Maintain a table $Q \colon S \times A \to \mathbb{R}$ (initialized to, e.g., 0)
- 2. Explore the environment according to π , resulting in a trajectory $s_0 a_1 s_1 a_2 s_2 \dots$
- 3. In step t, update Q by $Q(s_t, a_t) \leftarrow (1 - \alpha) \cdot Q(s_t, a_t) + \alpha [R(s_t, a_{t+1}, s_{t+1}) + \gamma \max_a Q(s_{t+1}, a)]$
- 4. After each episode, update π by $\pi(s, a) \leftarrow \arg\max_{a \in \mathcal{A}} Q(s, a)$
- 5. Repeat until this process converges; π is then the optimal policy

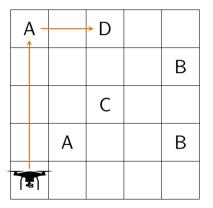
А		D	
			В
		С	
	А		В
	-		

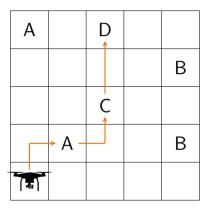


Daniel Neider: Reinforcement Learning with Reward Machines



Daniel Neider: Reinforcement Learning with Reward Machines

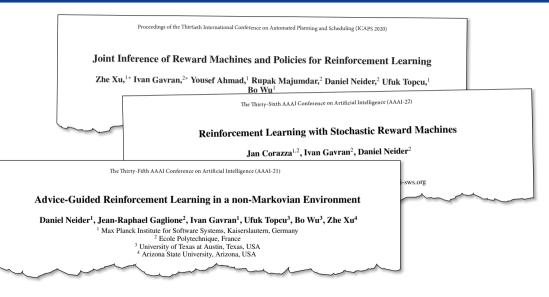




Α		D	
			В
		С	
	А		В
1-	-		

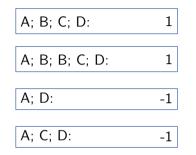
How to handle such situations?

Reinforcement Learning with Reward Machines



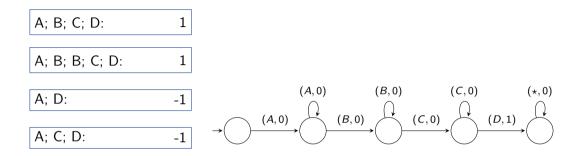
1. Joint Inference of Policies and Reward Machines

(joint work with Yousef Ahmad, Ivan Gavran, Rupak Majumdar, Ufuk Topcu, Bo Wu, and Zhe Xu)

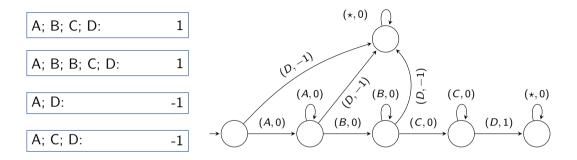


Bacchus et al. (1996) Jothimurgan et al. (2019) Icarte et al. (2018) Brafman et al. (2018)

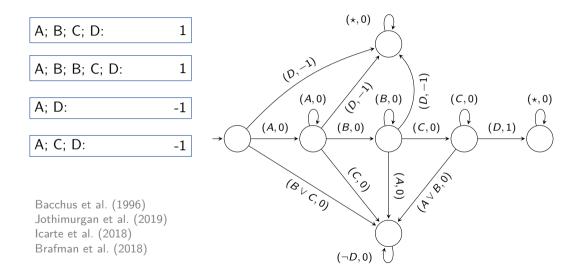
"Use automata/temporal logic to capture non-Markovian rewards"

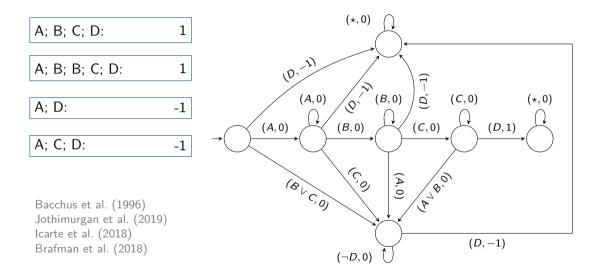


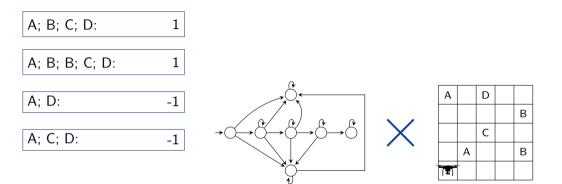
Bacchus et al. (1996) Jothimurgan et al. (2019) Icarte et al. (2018) Brafman et al. (2018)



Bacchus et al. (1996) Jothimurgan et al. (2019) Icarte et al. (2018) Brafman et al. (2018)







Bacchus et al. (1996) Jothimurgan et al. (2019) Icarte et al. (2018) Brafman et al. (2018) lcarte et al. (2018) have proposed an extension of the Q-learning algorithm, named QRM, that can handle reward machines

- avoids building the cross-product explicitly
- exploits the structure of the reward machine during exploration

Using Reward Machines for High-Level Task Specification and Decomposition in Reinforcement Learning

Rodrigo Toro Icarte^{1,2} Toryn Q. Klassen¹ Richard Valenzano³ Sheila A. McIlraith^{1,2}

Abstract

In this paper we propose Reward Machines - a type of finite state machine that supports the specification of reward functions while expening reing decomposition. We then present O-Learning for Reward Machines (ORM), an aleorithm which and uses off-policy q-learning to simultaneously learn subredicies for the different components. ORM is guaranteed to converge to an optimal policy in the tabular case, in contrast to Hierarchical Reinforcement Learning methods which might converse to subortimal policies. We demonstrate this behavior experimentally in two discrete demains. We also show how function approximation methods like neural networks can be incorporated into ORM, and that doing so can find better policies more quickly than hierarchical methods in a domain with a continuous state space.

1. Introduction

A startland assumption in conferencement learning (RL) in the that goard does not have access to the transmust model (Salan & Hans, 1999). This means that it does not know, the start of the start of the start of the start of the agent smatches and the start of the start of the start agent smatches the start of the start in the start of the start of the start of the start of the b hide the recent function from the agent. Artificial agent sames interpreter start from the transmission from the start of the start of the start of the start of the start agent start and the start of the start of the start of the start starts of the start of

¹Department of Computer Science, University of Toronto Toronto, Ontario, Canada ²Vector Institute, Toronto, Ontario Canada ³Element AI, Toronto, Ontario, Canada. Correspondence to: Radrigo-Toro Larte https://www.englistation.com Canada ³Element AI, Toronto, Ontario, Canada. Correspondence to: Radrigo-Toro Larte https://www.englistation.com Canada ³Element AI, Toronto, Ontario, Canada. Correspondence to: Radrigo-Toro Larte https://www.englistation.com Canada ³Element AI, Toronto, Ontario, Canada. Correspondence to: Radrigo-Toro Larte www.englistation.com Canada ³Element AI, Toronto, Ontario, Canada. Correspondence to: Radrigo-Toro Larte www.englistation.com Canada ³Element AI, Toronto, Ontario, Canada. Correspondence to: Radrigo-Toro Larte wwww.englistation.com Canada ³Element AI, Toronto, Ontario, Canada. Correspondence to: Radrigo-Toro Larte www.englistation.com Canada ³Element AI, Toronto, Ontario, Canada. Correspondence to: Radrigo-Toro Larte www.englistation.com Canada ³Element AI, Toronto, Ontario, Canada Correspondence to: Radrigo-Toro Larte www.englistation.com Correspondence Canada ³Element AI, Toronto, Conada ³Element AI, Toronto, Conada ³Element AI, Toronto, Canada ³

Proceedings of the 35th International Conference on Machine Learning, Stockholm, Sweden, PMLR 80, 2018. Copyright 2018 by the author(s). a programmed reward function is given as a black box to be agent. The agenc can query the function for the reward in the current situation, but does not have access to whatever structures or high-level idates the programmer may have used in defining it. However, an agent that had access to the specification of the reward function might be able to use it to decompose the problem and upond up learning. We comister a way to do so in this paper.

Previous work on giving an approximately as about the record function from send advanced as the specifications language, usually based on sub-goal sequences (Single, 2022), and in lane transport large (if circle, 2027). Cumuches et al., 2027), Laitmons et al., 2027, Timo Leante et al., 2021, Ritanabieg et al., 2021, and then generate a researd framition transmits hildling that specific ations. In this work, inclusions that engreges instructs the language, An such, sear approach is able to reward behaviors to varying degrees in amounts that cannot be expressed by previous approaches.

These are two main contributions of this work. First we introduce a type of finite state machine, called the Researd Mochine, which we use in defining rewards. A reward machine allows for composing different reward functions in flexihis ways, including concatenations, loops, and conditional rules. As an arent acts in the environment, merving from state to state, it also moves from state to state within a reward machine (as determined by high-level events detected within the environment). After every transition, the reward machine outputs the reward function the agent should use at that time. For example, we might construct a reward machine for "delivering coffee to an office" ming two states. In the first state, the second does not receive any sewards, but the second state, the secont nets rewards after delivering the coffee. The advantage of defining rewards this way is that the second knows that the problem consists of two states and might use this information for decomposing it.

Our second contribution is to introduce an algorithm, called Q-Learning for Researd Machines (QRM), that can exploit a reward machine's internal structure to decompose the problem and thereby improve sample efficiency. QRM's task decomposition does not prune optimal policies and uses q-

ICML 2018

lcarte et al. (2018) have proposed an extension of the Q-learning algorithm, named QRM, that can handle reward machines

- avoids building the cross-product explicitly
- exploits the structure of the reward machine during exploration

Problem

. . .

How does one construct reward machines?

direct construction, from temporal logics, learning,

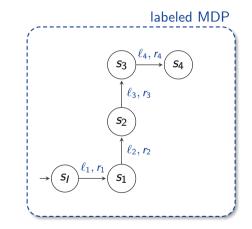
Using Reward Machines for High-Level Task Specification and Decomposition in Reinforcement Learning Rodrigo Toro Icarte¹² Toryn O. Klassen¹ Richard Valenzano³ Sheila A. McBraith¹¹ Abstract a reparamented research function is given as a black how to the anent. The anent can carry the function for the reward in In this paper we propose Reward Machines - a the current situation but does not have access to whatever type of finite state machine that supports the specstructures or high-level ideas the neueranneer may have ification of reward functions while expening reused in defining it. However, an agent that had access to the specification of the reward function might be able to use ing decomposition. We then present O-Learning it to decommone the problem and speed on learning. We for Reward Machines (ORM), an aleorithm which consider a way to do so in this maner Previous work on giving an agent knowledge about the and uses off-policy q-learning to simultaneously reward function focus on defining a task specification learn subredicies for the different components. ORM is guaranteed to converge to an optimal pollanguage, usually based on sub-goal sequences (Singh, 1992ach) or Inner temporal logic (Li et al. 2017; Comicy in the tabalar case, in contrast to Hierarchical cho et al., 2017; Littman et al., 2017; Toro Icarte et al., Reinforcement Learning methods which might 2018: Hausebois et al. 2018) and then neperate a researd converse to subortinal policies. We demonstrate function towards fulfilling that specification. In this work, this behavior experimentally in two discrete demains. We also show how function approximation not instead directly tackle the moblem of defining researd functions that expose structure to the agent. As such, our methods like neural networks can be incorporated approach is able to reward behaviors to varying degrees in into ORM, and that doing so can find better policies more quickly than hierarchical methods in a domain with a continuous state space. These are two main contributions of this work. First we introduce a type of finite state machine, called the Researd Mochine, which we use in defining rewards. A reward machine 1. Introduction allows for composing different reward functions in flexi-A standard assumption in reinforcement learning (RL) is his ways, including concatenations, loops, and conditional rules. As an arent acts in the environment, merving from that the second does not have access to the emirorment model state to state, it also moves from state to state within a re-(Sutton & Barto, 1998). This means that it does not know, a ward machine (as determined by high-level events detected priori the transition probabilities or presend function moniwithin the environment). After every transition, the reward fest in the environment. To learn optimal behavior, an RL machine outputs the reward function the agent should use at that time. For example, we might construct a reward from its enuminese. While according that the termitian medmachine for "delivering coffee to an office" ming two states. abilities are antenews accors reasonable, there is less reason In the first state, the second does not receive any sewards, but to hide the second function from the secont. Astificial second cannot inherently perceive reward from the environment; common must prove three many liter the trent interthe second state, the secont nets rewards after delivering the coffee. The advantage of defining rewards this way is that arent is interacting with the real world). Typically, though the second knows that the resolution containts of two states and Department of Computer Science, University of Taronto, might use this information for decomposing it. Toronto, Ontario, Canada ²Vector Institute, Toronto, Ontario Our second contribution is to introduce an algorithm, called ty Badrina Tara Larte conterrolles terrorte ele-O.L coming for Record Machines (ORM), that can earlight a reward machine's internal structure to decompose the prob-Proceedings of the 35th International Conference on Machine lem and thereby improve sample efficiency. ORM's task Learning, Stockholm, Sweden, PMLR 80, 2018, Corveright 2018. decomposition does not more optimal policies and mes a-

ICML 2018

Joint Inference of Policies and Reward Machines

Key idea

- Given the current hypothesis reward machine *H*, perform QRM and record the resulting label sequence λ = ℓ₁...ℓ_n and reward sequence ρ = r₁...r_n
- If the pair (λ, ρ) contradicts H, learn a new reward machine H'
- Repeat until this process converges to the "true" reward machine and an optimal policy



Initialize reward machine H;

(*,0) *H*: $\rightarrow (p_1)$

Initialize reward machine H;

Initialize a set Q of q-functions;

$$H: \longrightarrow \begin{array}{c} & & \\$$

 $Q: \{q^{p_1}\}$

Initialize reward machine H;

Initialize a set Q of q-functions;

Initialize a sample X of traces;

H: (*,0) $\rightarrow (p_1$

 $Q: \qquad \{q^{p_1}\}$

X: Ø

Initialize H, Q, X;

repeat

$$(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$$

$$H: \longrightarrow \stackrel{P_1}{\longrightarrow} (\star, 0)$$

 $Q: \qquad \left\{q^{p_1}\right\}$

X: ∅

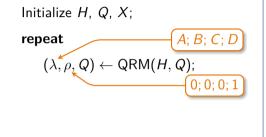
Initialize H, Q, X;
repeat
$$A; B; C; D$$

 $(\lambda, \rho, Q) \leftarrow QRM(H, Q);$

$$H: \longrightarrow p_1 (\star, 0)$$

 $Q: \{q^{p_1}\}$

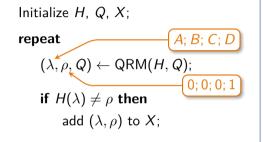
X: ∅



H: (*,0) $\rightarrow (p_1)$

 $Q: \{q^{p_1}\}$

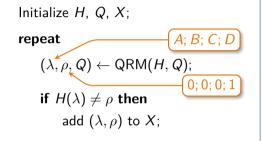
X: ∅



$$H: \rightarrow \begin{array}{c} \rightarrow \\ P_1 \\ \leftarrow \end{array} (\star, 0)$$

 $Q: \qquad \left\{q^{p_1}\right\}$

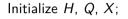
X: ∅



$$H: \longrightarrow \stackrel{p_1}{\longrightarrow} (\star, 0)$$

Q: $\{q^{p_1}\}$

 $X: \{ (A; B; C; D/0; 0; 0; 1) \}$



repeat

 $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$

 $\begin{array}{l} \text{if } H(\lambda) \neq \rho \text{ then} \\ \quad \text{add } (\lambda, \rho) \text{ to } X; \end{array}$

 $\begin{array}{l} \text{if } X \text{ was modified then} \\ H \leftarrow \operatorname{infer}(X); \end{array}$

Initialize H, Q, X;

repeat

- $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$
- $\begin{array}{l} \text{if } H(\lambda) \neq \rho \text{ then} \\ \quad \text{add } (\lambda, \rho) \text{ to } X; \end{array}$
- $\begin{array}{l} \text{if } X \text{ was modified then} \\ H \leftarrow \operatorname{infer}(X); \end{array}$

- $(\neg D, 0)$ $(\neg D, 1)$ (P_1) $(\star, 0)$ $Q: \{q^{p_1}\}$
 - $X: \{ (A; B; C; D/0; 0; 0; 1) \}$

Initialize H, Q, X;

repeat

- $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$
- $\begin{array}{l} \text{if } \mathcal{H}(\lambda) \neq \rho \text{ then} \\ \\ \text{add } (\lambda, \rho) \text{ to } X; \end{array} \end{array}$
- if X was modified then $H \leftarrow infer(X);$ re-initialize Q if necessary;

 $(\neg D, 0)$ (D, 1)H: p_2 $(\star, 0)$ $\{q^{p_1}\}$ Q: $\{(A; B; C; D/0; 0; 0; 1)\}$ X

Initialize H, Q, X;

repeat

- $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$
- $\begin{array}{l} \text{if } H(\lambda) \neq \rho \text{ then} \\ \quad \text{add } (\lambda, \rho) \text{ to } X; \end{array}$
- if X was modified then $H \leftarrow infer(X);$ re-initialize Q if necessary;

$$H: \longrightarrow \stackrel{(\neg D, 0)}{\underset{p_1}{\longrightarrow} (D, 1)} \xrightarrow{p_2} Q: \qquad \{q^{p_1}, q^{p_2}\}$$
$$X: \qquad \{(A; B; C; D/0; 0; 0; 1)\}$$

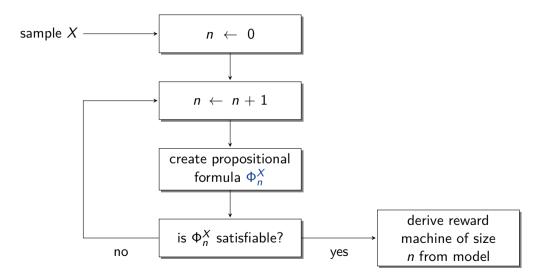
Initialize H, Q, X;

repeat

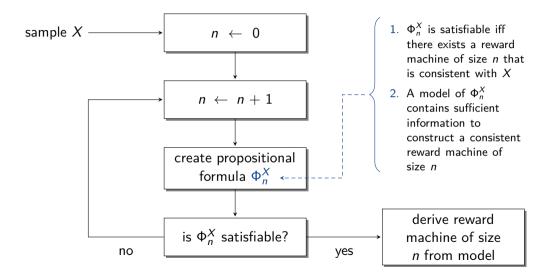
- $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$
- $\begin{array}{l} \text{if } \mathcal{H}(\lambda) \neq \rho \text{ then} \\ \quad \text{add } (\lambda, \rho) \text{ to } X; \end{array}$
- if X was modified then $H \leftarrow infer(X);$ re-initialize Q if necessary;

It is crucial to infer minimal reward machines

A SAT-Based Inference Algorithm for Reward Machines



A SAT-Based Inference Algorithm for Reward Machines



We use two sets of propositional variables to encode reward machines:

- $d_{p,\ell,q}$ encodes the transition function of the reward machine (i.e., the machine transitions from state p to state q on reading symbol ℓ)
- $o_{p,\ell,r}$

encodes the output function of the reward machine (i.e., the machine outputs reward r in state p on reading symbol ℓ)

We use two sets of propositional variables to encode reward machines:

- $d_{p,\ell,q}$ encodes the transition function of the reward machine (i.e., the machine transitions from state p to state q on reading symbol ℓ)
- $o_{p,\ell,r}$ encodes the output function of the reward machine (i.e., the machine outputs reward r in state p on reading symbol ℓ)

Enforcing deterministic functions

We impose pseudo-Boolean constraints to enforce for each pair of state p and input a that

- exactly one variable $d_{p,\ell,q}$ is set to true
- exactly one variable $o_{p,\ell,r}$ is set to true

 $x_{\lambda,p}$ encodes the run of the reward machine on all prefixes of examples (i.e., the machine reaches state *p* after reading the prefix λ)

 $x_{\lambda,p}$ encodes the run of the reward machine on all prefixes of examples (i.e., the machine reaches state *p* after reading the prefix λ)

Enforcing consistency with the examples

$$\begin{bmatrix}\bigwedge_{u \in Pref(X)} \operatorname{one}(x_{u,q_1}, \dots, x_{u,q_n}) \end{bmatrix} \land x_{\varepsilon,q_l}$$
$$(x_{\lambda,p} \land d_{p,\ell,q}) \to x_{\lambda\ell,q}$$
$$x_{\lambda,p} \to o_{p,\ell,r}$$

reward machine:

$$\rightarrow q_l$$

Daniel Neider: Reinforcement Learning with Reward Machines

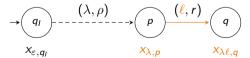
12

 $x_{\lambda,p}$ encodes the run of the reward machine on all prefixes of examples (i.e., the machine reaches state *p* after reading the prefix λ)

Enforcing consistency with the examples

$$\left[\bigwedge_{u \in Pref(X)} \operatorname{one}(x_{u,q_1}, \dots, x_{u,q_n})\right] \wedge x_{\varepsilon,q_l}$$
$$(x_{\lambda,p} \wedge d_{p,\ell,q}) \to x_{\lambda\ell,q}$$
$$x_{\lambda,p} \to o_{p,\ell,r}$$

reward machine:

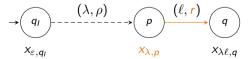


 $x_{\lambda,p}$ encodes the run of the reward machine on all prefixes of examples (i.e., the machine reaches state *p* after reading the prefix λ)

Enforcing consistency with the examples

$$\left[\bigwedge_{u \in Pref(X)} \operatorname{one}(x_{u,q_1}, \dots, x_{u,q_n})\right] \wedge x_{\varepsilon,q_l}$$
$$(x_{\lambda,p} \wedge d_{p,\ell,q}) \to x_{\lambda\ell,q}$$
$$x_{\lambda,p} \to o_{p,\ell,r}$$

reward machine:



Theorem (Ahmad, Gavran, Majumdar, N., Topcu, Wu, and Xu)____

Given

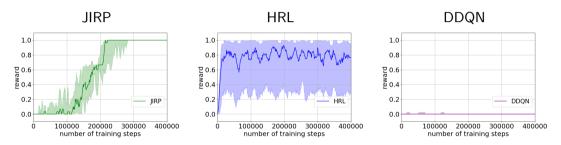
- > a sufficient episode length
- an ε-greedy exploration strategy

we have the following:

- 1. JIRP almost surely learns the "true" reward machine
- 2. JIRP almost surely converges to an optimal policy

JIRP: Empirical Results

Office World Scenario (Icarte et al., 2018)

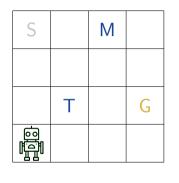


Conclusion

- JIRP is the only method that converges to an optimal policy
- ▶ JIRP converges faster than any of the competing methods

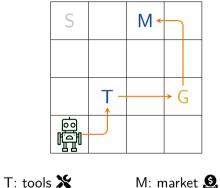
2. Reinforcement Learning with Stochastic Reward Machines

(joint work with Jan Corazza and Ivan Gavran)

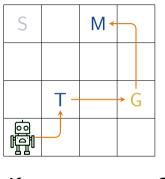




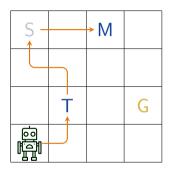






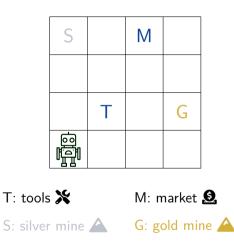


T: tools 🗙	M: market 🗕
S: silver mine 🔺	G: gold mine 🔺



T: tools X	M: market 🗕
S: silver mine 🔺	G: gold mine 🔺

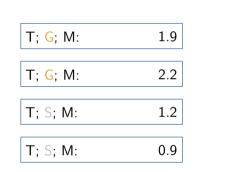
T; <mark>G</mark> ; M:	1.9
T; <mark>G</mark> ; M:	2.2
T; S; M:	1.2
T; S; M:	0.9

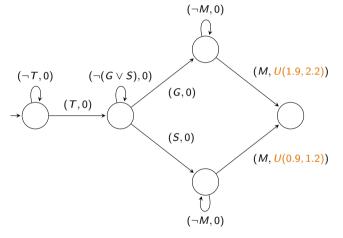


T; G; M:	1.9
T; <mark>G</mark> ; M:	2.2
T; S; M:	1.2
T; S; M:	0.9

- If the label sequences are identical, no reward machine matches both traces
- If the label sequences are different, the resulting reward machine explodes in size

Stochastic Reward Machines (SRM)

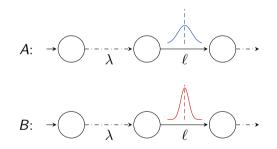




Outputs are bounded continuous distributions

Equivalence in Expectation

Two SRMs A and B are equivalent in expectation $(A \sim_E B)$ if they output sequences of distributions with equal expected values for each label sequence

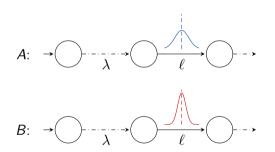


Equivalence in Expectation

Two SRMs A and B are equivalent in expectation $(A \sim_E B)$ if they output sequences of distributions with equal expected values for each label sequence

Corollary

If two SRMs are equivalent in expectation, then they induce the same optimal policy in an environment



A Naive Solution

A naive algorithm

- 1. Collect many samples
- 2. Take the average reward in every position of the same trajectory
- 3. Construct an ordinary reward machine based on the average rewards

T; <mark>G</mark> ; M:	1.9
T; G; M:	2.2
T; G; M:	2.1
T; G; M:	2.0
	average: 2.05

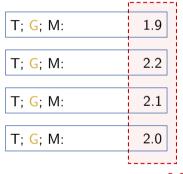
A Naive Solution

A naive algorithm

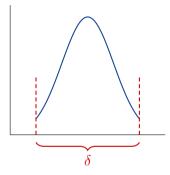
- 1. Collect many samples
- 2. Take the average reward in every position of the same trajectory
- 3. Construct an ordinary reward machine based on the average rewards

Problem

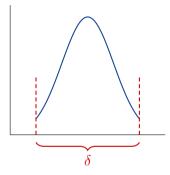
Collecting samples is too slow!



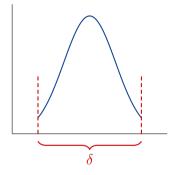
average: 2.05



1. Probability distributions are continuous and have bounded support with "width" δ

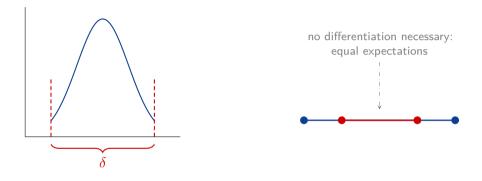


- 1. Probability distributions are continuous and have bounded support with "width" δ
- The noise from one distribution does not fully conceal the signal from another one (except in symmetric circumstances)



we will eventually observe enough rewards to differentiate

- 1. Probability distributions are continuous and have bounded support with "width" δ
- 2. The noise from one distribution does not fully conceal the signal from another one (except in symmetric circumstances)



- 1. Probability distributions are continuous and have bounded support with "width" δ
- The noise from one distribution does not fully conceal the signal from another one (except in symmetric circumstances)



Initialize H, Q, X, A;

repeat

 $(\lambda, \rho, Q) \leftarrow QRM(H, Q);$ add (λ, ρ) to A; _______ one would keep a moving average in practice



Initialize H, Q, X, A;

repeat

 $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$ add (λ, ρ) to A;

if H is not δ -consistent with (λ, ρ) then add (λ, ρ) to X; $H' \leftarrow infer(X)$; infers a minimal δ -consistent "proto"-SRM

(only cares for δ -consistency, not estimating distribution)



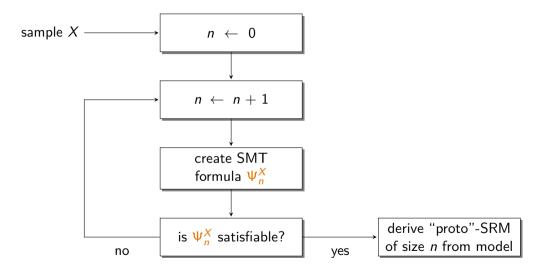
Initialize H, Q, X, A;

repeat

 $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$ add (λ, ρ) to A;

if H is not δ -consistent with (λ, ρ) then add (λ, ρ) to X; $H' \leftarrow infer(X)$; $H \leftarrow estimate(H', A)$; re-initialize Q if necessary; corrects outputs of H'
by estimating distribution parameters from samples in A

An SMT-Based Inference Algorithm for Stochastic Reward Machines



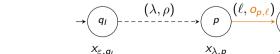
We use propositional and real-valued variables to encode a "proto"-SRM:

- $d_{p,\ell,q} \in \mathbb{B}$ encodes the transition function of the reward machine
- $x_{\lambda,p} \in \mathbb{B}$ encodes the run of the reward machine on prefixes from X
- $o_{p,\ell} \in \mathbb{R}$ encodes a "conjectured mean" of an output distribution (i.e., the distr. returned in state p on reading symbol ℓ has mean $o_{p,\ell}$)

Enforcing consistency with the examples

 $|x_{\lambda,p} \to |o_{p,\ell} - r| \le \frac{\delta}{2}$

SRM:



q

 $X_{\lambda \ell, a}$

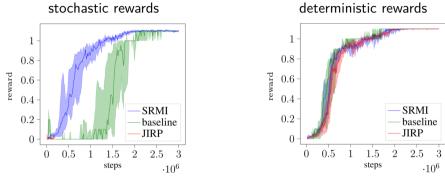
Theorem (Corazza, Gavran, N.)

Given

- a sufficient episode length
- an ε-greedy exploration strategy
- ► Assumptions 1 and 2 hold for the "true" (environment) SRM we have the following:
 - 1. SRMI almost surely learns a SRM that is equivalent in expectation to the "true" SRM
 - 2. SRMI almost surely converges to an optimal policy

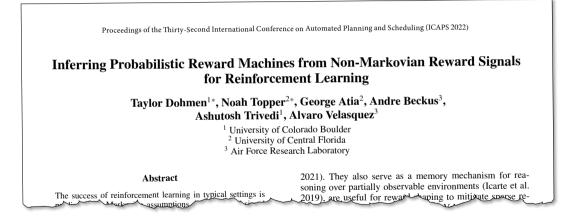
SRMI: Empirical Results

Mining Environment



Conclusion

- SRMI converges faster than the baseline method
- SRMI's performance does not degrade in the case of deterministic rewards



3. Advice-Guided Reinforcement Learning

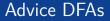
(joint work with Jean-Raphaël Gaglione, Ivan Gavran, Ufuk Topcu, Bo Wu, and Zhe Xu)

Advice-Guided Reinforcement Learning

Xu et al., 2020







We formalize advice by means of regular languages:

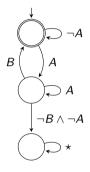
- Deterministic Finite Automata (DFA)
- Regular expressions
- Linear Temporal Logic
- . . .

Advice DFAs

We formalize advice by means of regular languages:

- Deterministic Finite Automata (DFA)
- Regular expressions
- Linear Temporal Logic

...



"every A is followed by B"

Advice DFAs

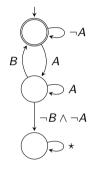
We formalize advice by means of regular languages:

- Deterministic Finite Automata (DFA)
- Regular expressions
- Linear Temporal Logic
- ► ..

Compatibility of advice DFAs (i.e., semantics)

A reward can only be positive (negative/non-zero) if the advice DFA accepts the label sequence

 A reward machine satisfying this property is called compatible



"every A is followed by B"

$H: \rightarrow (p_1) (\star, 0)$

Initialize reward machine H;

Initialize reward machine H;

Initialize a set Q of q-functions;

$$H: \rightarrow \begin{array}{c} & & \\$$

 $Q: \{q^{p_1}\}$

Initialize reward machine H;

Initialize a set Q of q-functions;

Initialize a sample X of traces;

$$H: \rightarrow \begin{array}{c} \rightarrow \\ P_1 \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \end{array} (*, 0)$$

 $Q: \qquad \left\{q^{p_1}\right\}$

B

В

D

Initialize reward machine H;

Initialize a set Q of q-functions;

Initialize a sample X of traces;

Initialize a set D of advice DFAs;

$$H: \rightarrow \overbrace{P_{1}}^{P_{1}} (\star, 0)$$

$$Q: \{q^{P_{1}}\}$$

$$X: \emptyset$$

$$\xrightarrow{\neg A} \qquad A \qquad \bigcirc \qquad \neg B \qquad \bigcirc$$

R

D:

Initialize H, Q, X, D;

repeat

 $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$

$$H: \rightarrow \overbrace{P_{1}}^{p_{1}} (\star, 0)$$

$$Q: \{q^{p_{1}}\}$$

$$X: \emptyset$$

$$D: \left\{ \rightarrow \overbrace{B}^{A} \xrightarrow{A} \xrightarrow{A} \xrightarrow{\neg B} \xrightarrow{B} \xrightarrow{B} \xrightarrow{B} \xrightarrow{A} \xrightarrow{P_{1}} \xrightarrow{P_{2}} \xrightarrow{P_{2}}$$

Initialize *H*, *Q*, *X*, *D*;
repeat
$$A; B; C; D$$

 $(\lambda, \rho, Q) \leftarrow QRM(H, Q);$

$$H: \rightarrow \stackrel{p_1}{\longrightarrow} (\star, 0)$$

$$Q: \{q^{p_1}\}$$

$$X: \emptyset$$

$$D: \left\{ \rightarrow \stackrel{\neg A}{\longrightarrow} \stackrel{A}{\longrightarrow} \stackrel{\neg B}{\longrightarrow} \stackrel{B}{\longrightarrow} \stackrel{P}{\longrightarrow} \right\}$$

Initialize H, Q, X, D;
repeat
$$A; B; C; D$$

 $(\lambda, \rho, Q) \leftarrow QRM(H, Q);$
 $0; 0; 0; 1$

$$H: \rightarrow \stackrel{p_1}{\longrightarrow} (\star, 0)$$

$$Q: \{q^{p_1}\}$$

$$X: \emptyset$$

$$D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \stackrel{A}{\bigoplus} \stackrel{\frown}{\bigoplus} \stackrel{\neg B}{\bigoplus} \stackrel{B}{\bigoplus} \stackrel{\frown}{\bigoplus} \}$$

Initialize *H*, *Q*, *X*, *D*;
repeat
$$A; B; C; D$$

 $(\lambda, \rho, Q) \leftarrow QRM(H, Q);$
if $H(\lambda) \neq \rho$ then
add (λ, ρ) to *X*;

$$H: \rightarrow \stackrel{p_1}{\longrightarrow} (\star, 0)$$

$$Q: \{q^{p_1}\}$$

$$X: \emptyset$$

$$D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \stackrel{A}{\bigoplus} ; \rightarrow \stackrel{\neg B}{\bigoplus} \stackrel{B}{\bigoplus} \\ D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \stackrel{A}{\bigoplus} ; \rightarrow \stackrel{\neg B}{\bigoplus} \stackrel{B}{\bigoplus} \\ D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \stackrel{A}{\bigoplus} ; \rightarrow \stackrel{\neg B}{\bigoplus} \stackrel{B}{\bigoplus} \\ D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \stackrel{A}{\bigoplus} ; \rightarrow \stackrel{\neg B}{\bigoplus} \stackrel{B}{\bigoplus} \\ D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \stackrel{A}{\bigoplus} ; \rightarrow \stackrel{\neg B}{\bigoplus} \stackrel{B}{\bigoplus} \\ D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \stackrel{A}{\bigoplus} ; \rightarrow \stackrel{\neg B}{\bigoplus} \stackrel{B}{\bigoplus} \\ D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \stackrel{A}{\bigoplus} ; \rightarrow \stackrel{\neg B}{\bigoplus} \\ D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \stackrel{A}{\bigoplus} \\ D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \stackrel{A}{\bigoplus} \\ D: \{ \rightarrow \stackrel{\neg A}{\bigoplus} \\ D: \{ \rightarrow \stackrel{\cap A}$$

Initialize H, Q, X, D;
repeat A; B; C; D

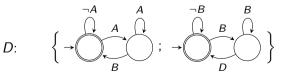
$$(\lambda, \rho, Q) \leftarrow QRM(H, Q);$$

if $H(\lambda) \neq \rho$ then
add (λ, ρ) to X;

$$H: \longrightarrow p_1 (\star, 0)$$

 $Q: \{q^{p_1}\}$

 $X: \{ (A; B; C; D/0; 0; 0; 1) \}$



Initialize *H*, *Q*, *X*, *D*;
repeat

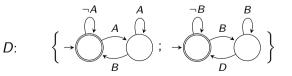
$$(\lambda, \rho, Q) \leftarrow QRM(H, Q);$$

if $H(\lambda) \neq \rho$ then
add (λ, ρ) to *X*;

if (λ, ρ) is not compatible with some $\mathcal{D} \in D$ then remove \mathcal{D} from D; $H: \rightarrow \begin{array}{c} \rightarrow \\ p_1 \\ \leftarrow \end{array} (\star, 0)$

 $Q: \{q^{p_1}\}$

 $X: \{ (A; B; C; D/0; 0; 0; 1) \}$



Initialize *H*, *Q*, *X*, *D*;
repeat

$$(\lambda, \rho, Q) \leftarrow QRM(H, Q);$$

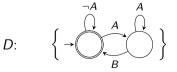
if $H(\lambda) \neq \rho$ then
add (λ, ρ) to *X*;

if (λ, ρ) is not compatible with some $\mathcal{D} \in D$ then remove \mathcal{D} from D;

$$H: \rightarrow \begin{array}{c} \rightarrow \\ p_1 \\ \hline \end{array} (\star, 0)$$

 $Q: \{q^{p_1}\}$

$$X: \{ (A; B; C; D/0; 0; 0; 1) \}$$



Initialize H, Q, X, D;

repeat

- $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$
- if $H(\lambda) \neq \rho$ then add (λ, ρ) to X;
- $\begin{array}{l} \text{if } (\lambda,\rho) \text{ is not compatible with} \\ \text{some } \mathcal{D} \in D \text{ then} \\ \\ \text{remove } \mathcal{D} \text{ from } D; \end{array}$
- if X or D were modified then $H \leftarrow infer(X, D);$

28



Initialize H, Q, X, D;

repeat

- $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$
- if $H(\lambda) \neq \rho$ then add (λ, ρ) to X;
- $\begin{array}{l} \textbf{if} \ (\lambda,\rho) \ \textbf{is not compatible with} \\ \textbf{some } \mathcal{D} \in D \ \textbf{then} \\ \\ \textbf{remove } \mathcal{D} \ \textbf{from } D; \end{array}$
- if X or D were modified then $H \leftarrow infer(X, D);$

$$(\neg A, 0) \qquad (\neg D, 0)$$

$$(\neg D, 0) \qquad (\neg D, 0)$$

$$(\neg D, 0) \qquad (\neg D, 0)$$

$$(P_{2} \qquad (B, 0) \qquad (P_{3} \qquad (D, 1))$$

$$Q: \qquad \{q^{p_{1}}\}$$

$$X: \qquad \{(A; B; C; D/0; 0; 0; 1)\}$$

$$D: \qquad \left\{ \rightarrow \bigcirc A \qquad A \\ B \qquad (P_{3} \qquad (P_{3})) \\ (D, 1) \qquad (P_{3}) \qquad (P_{3}) \\ (D, 1) \qquad (P_{3}) \qquad (P_{3}) \\ (D, 1) \qquad (P_{3}) \qquad (P_{3}) \\ (D, 1) \qquad (P_{3}) \qquad (P_{3}) \\ (D, 1) \qquad (P_{3}) \qquad (P_{3}) \\ (D, 1) \qquad (P_{3}) \qquad (P_{3}) \\ (D, 1) \qquad (P_{3}) \qquad (P_{3}) \qquad (P_{3}) \\ (D, 1) \qquad (P_{3}) \qquad (P_{3}) \qquad (P_{3}) \\ (D, 1) \qquad (P_{3}) \qquad (P_{3}) \qquad (P_{3}) \\ (D, 1) \qquad (P_{3}) \qquad (P_{$$



Initialize H, Q, X, D;

repeat

- $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$
- if $H(\lambda) \neq \rho$ then add (λ, ρ) to X;
- $\begin{array}{l} \textbf{if} \ (\lambda,\rho) \ \textbf{is not compatible with} \\ \textbf{some } \mathcal{D} \in D \ \textbf{then} \\ \\ \textbf{remove } \mathcal{D} \ \textbf{from } D; \end{array}$
- if X or D were modified then $H \leftarrow infer(X, D);$ re-initialize Q if necessary;—

$$H: \xrightarrow{(\neg A, 0)} (\neg D, 0)$$

$$H: \xrightarrow{(A, 0)} (P_2) (B, 0) (P_3) (D, 1)$$

$$\Rightarrow Q: \{q^{p_1}\}$$

$$X: \{(A; B; C; D/0; 0; 0; 1)\}$$

$$D: \{\overrightarrow{A, A}, A$$

$$B \}$$



Initialize H, Q, X, D;

repeat

- $(\lambda, \rho, Q) \leftarrow \mathsf{QRM}(H, Q);$
- if $H(\lambda) \neq \rho$ then add (λ, ρ) to X;
- $\begin{array}{l} \textbf{if} \ (\lambda,\rho) \ \textbf{is not compatible with} \\ \textbf{some } \mathcal{D} \in D \ \textbf{then} \\ \\ \textbf{remove } \mathcal{D} \ \textbf{from } D; \end{array}$
- if X or D were modified then $H \leftarrow infer(X, D);$ re-initialize Q if necessary;—

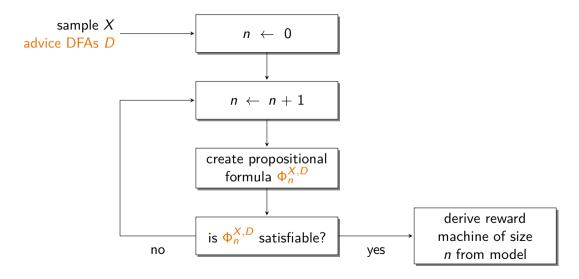
$$H: \xrightarrow{(\neg A, 0)} \xrightarrow{(\neg D, 0)} \xrightarrow{(\neg D, 0)} \xrightarrow{(P_1)} \xrightarrow{(A, 0)} \xrightarrow{(P_2)} \xrightarrow{(B, 0)} \xrightarrow{(P_3)} \xrightarrow{(D, 1)}$$

$$Q: \{q^{p_1}, q^{p_2}, q^{p_3}\}$$

$$X: \{(A; B; C; D/0; 0; 0; 1)\}$$

$$D: \{\overrightarrow{\neg A} \qquad A \\ \overrightarrow{\bigcirc} \qquad \overrightarrow{B} \}$$

A SAT-Based Inference Algorithm for Reward Machines



Theorem (N., Gaglione, Gavran, Topcu, Wu, Xu)_

Given

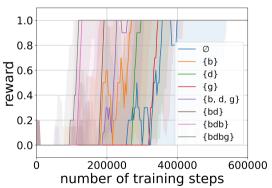
- > a sufficient episode length
- an ε-greedy exploration strategy

we have the following:

- 1. AdvisoRL almost surely learns the "true" reward machine
- 2. AdvisoRL almost surely converges to an optimal policy

AdvisoRL: Empirical Results

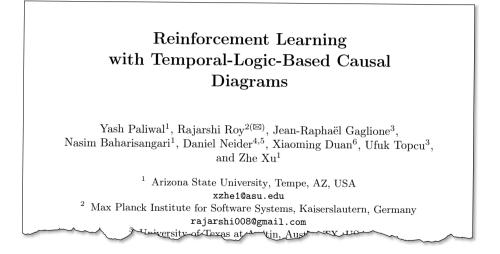
Office World Scenario (Icarte et al., 2018) AdvisoRL



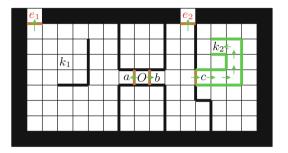
Conclusion

- AdvisoRL's performance improves with the "quality" of the given advice
- AdvisoRL is robust to incorrect advice





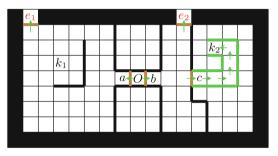
Grid World Example



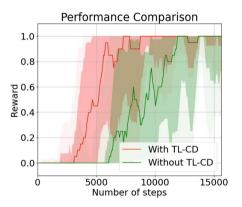
$$b \models \mathsf{G} \neg e_1$$
$$c \models \mathsf{X} \mathsf{X} \mathsf{X} \mathsf{X} \mathsf{K}_2$$
$$k_2 \models \mathsf{G} \neg e_2$$

Daniel Neider: Reinforcement Learning with Reward Machines

Grid World Example



$$b \models \mathsf{G} \neg e_1$$
$$c \models \mathsf{X} \mathsf{X} \mathsf{X} \mathsf{X} \mathsf{X} k_2$$
$$k_2 \models \mathsf{G} \neg e_2$$



Summary

- ▶ We have been on a journey through reinforcement learning with reward machines
- There are several extension (often by other research groups)
 - partial observability, active automata learning, etc.

Future work

- Incorporating (temporal) causal information
- Automatically synthesizing high level propositions
- More expressive classes of finite-state machines (e.g., counter)

Newly Established Research Center at UA Ruhr



- Three universities: University of Duisburg-Essen, University of Bochum, TU Dortmund University
- **Four disciplines:** Computer science, IT Security, Statistics, Psychology

We offer opportunities ...

- Collaborations with academia and industry
- Open positions for research group leaders, postdocs, Ph.D.s, students
- Internship program