Concurrent Stochastic Lossy Channel Games

Daniel Stan (EPITA)

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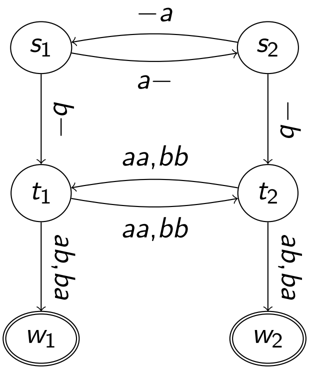
joint work with **Anthony W. Lin** (MPI-SWS, RPTU),
Muhammad Najib (Heriot-Watt) and
Parosh Abdullah (Uppsala University)

Outline

1. Finite Concurrent Games
2. Lossy Channel Systems
3. Infinite Concurrent Games

Stochastic Concurrent Finite Games

Concurrent Game on a Finite graph



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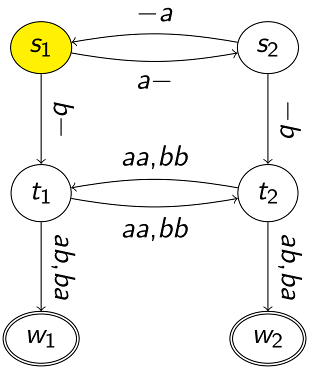

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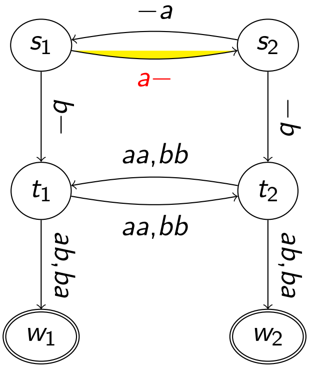

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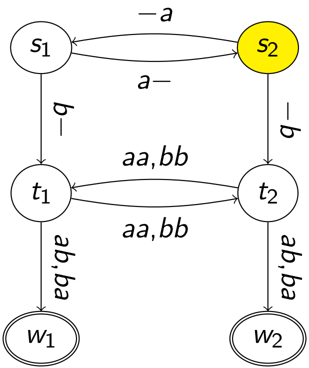

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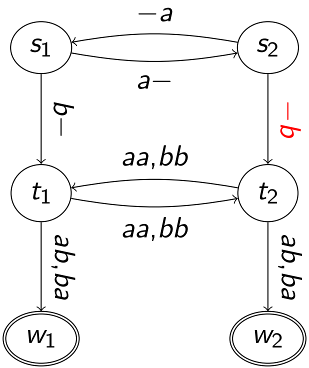

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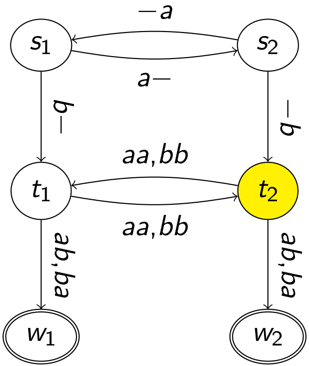

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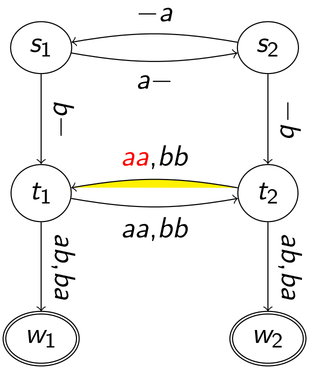

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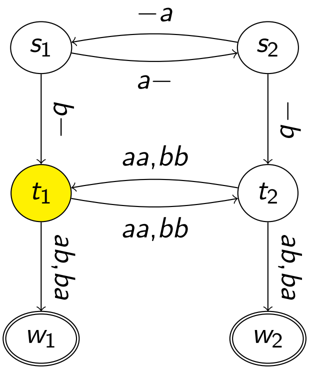

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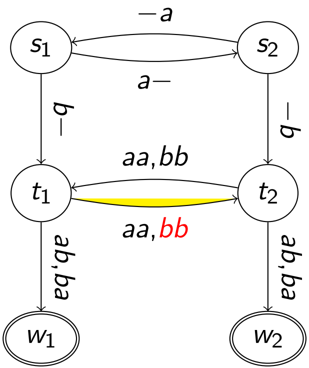

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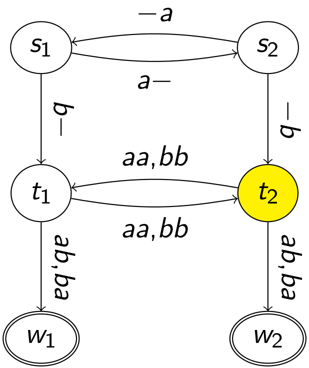

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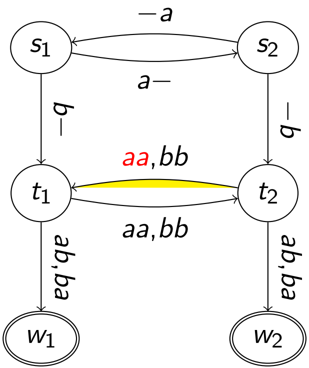

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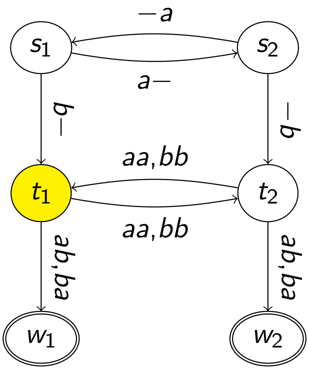

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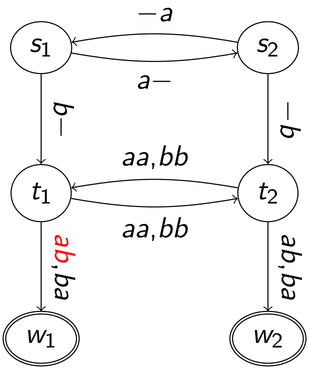

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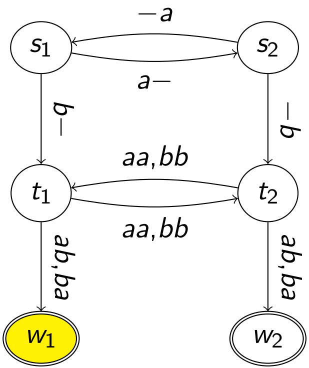

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- Several agents
- For each state $s \in S$, each $i \in \mathrm{Agt}$, set of $\mathrm{Act}_i(s)$ actions
- Transitions played concurrently

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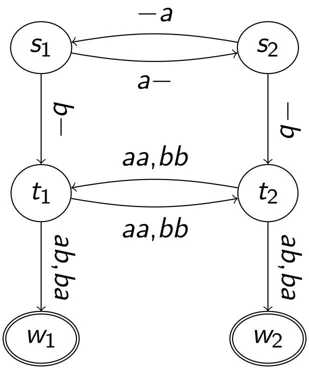

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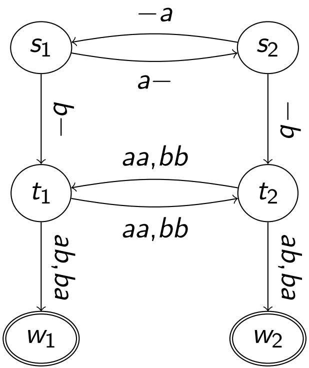

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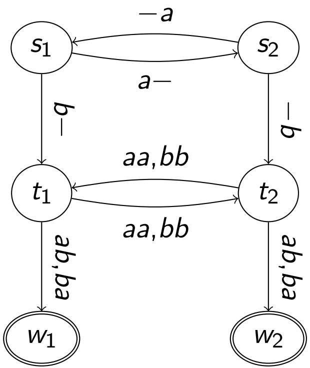

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  (Example here: $\Diamond w_1 \not\equiv \neg \Diamond w_2$ so not zero-sum)

Strategy Classes

A strategy for player i is:

$$\sigma_i : S^+ \rightarrow \mathrm{Dist}(\mathrm{Act})$$

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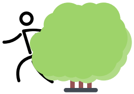
Can't we just play with **DP** strategies only?

Skirmish Game [dAHK07]



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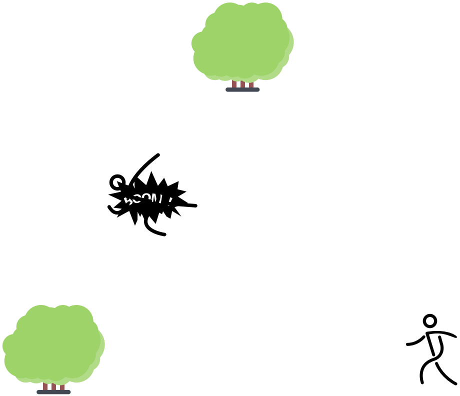

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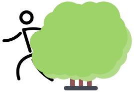

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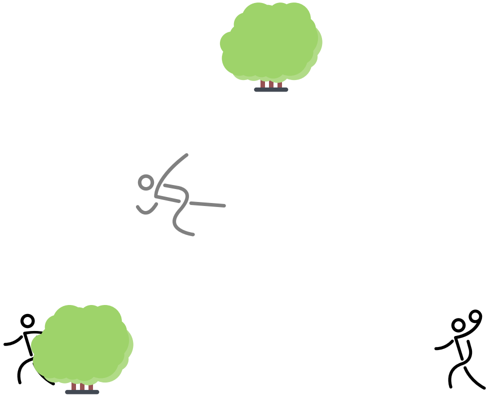

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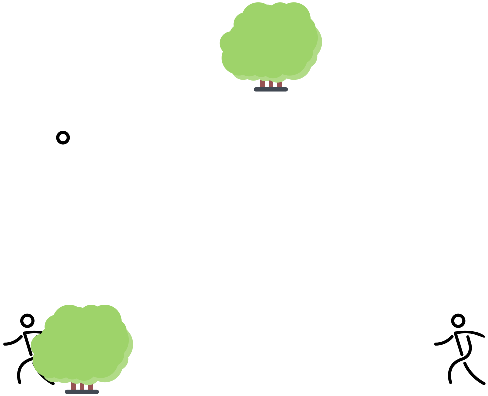

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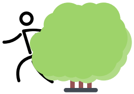

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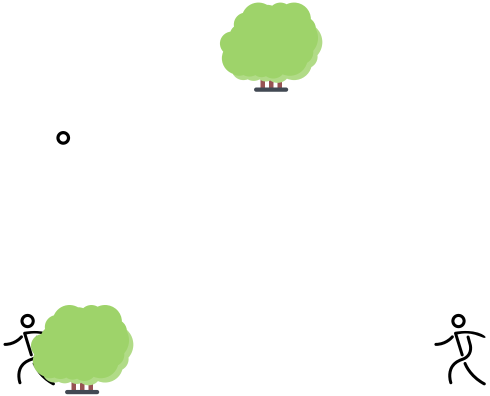

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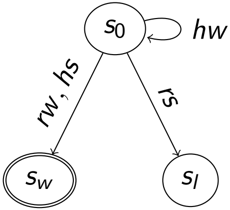

P1: **reach with probability** 1
aka $\mathrm{AS}(\Diamond R)$
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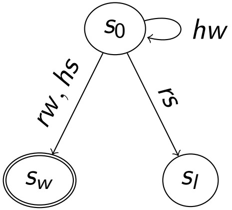

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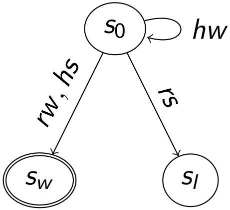

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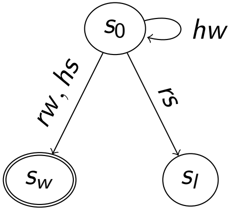

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Still, **Player 2** wins this game but with an **infinite memory strategy**.

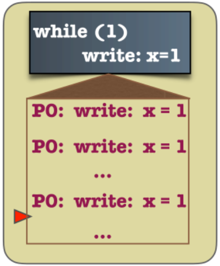
$$\forall n, \sigma_2(s_0^n)[s] = \left(\frac{1}{2}\right)^{2^{-n}}$$

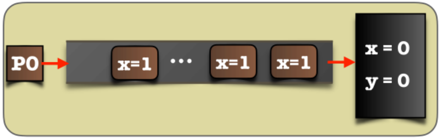
Channel Systems

Channel Systems (FIFO): Motivations

Modelisation and verification of systems with:

- Network transmissions;
- Transactional operations;
- TSO semantics [AABN18]





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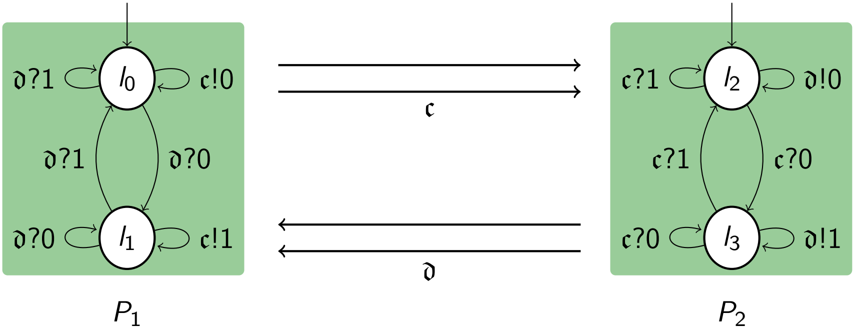

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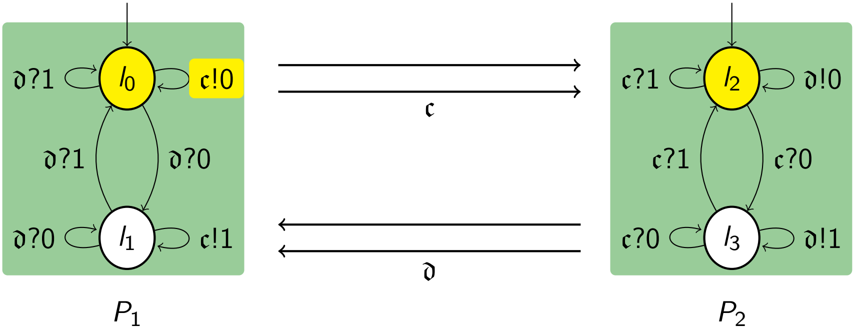

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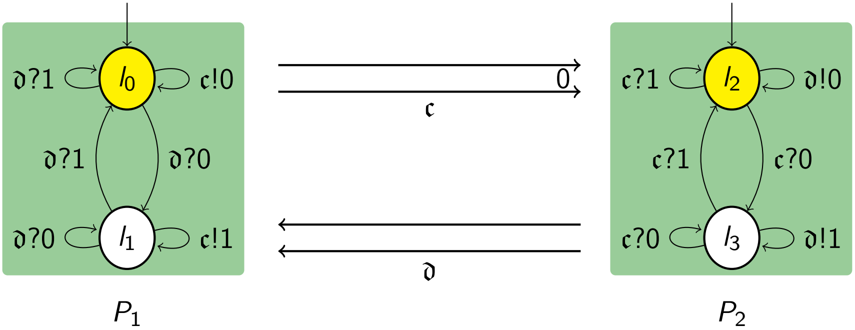

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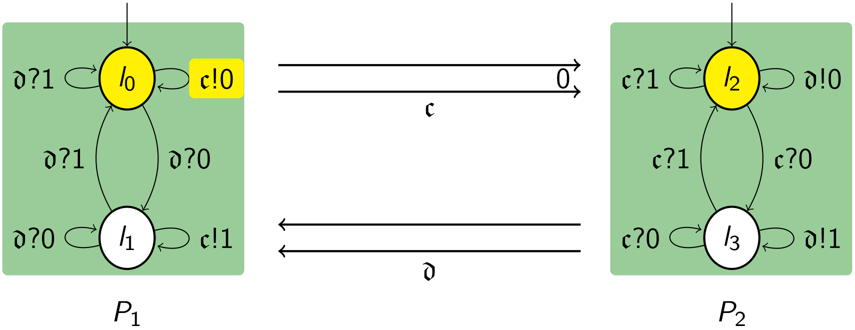

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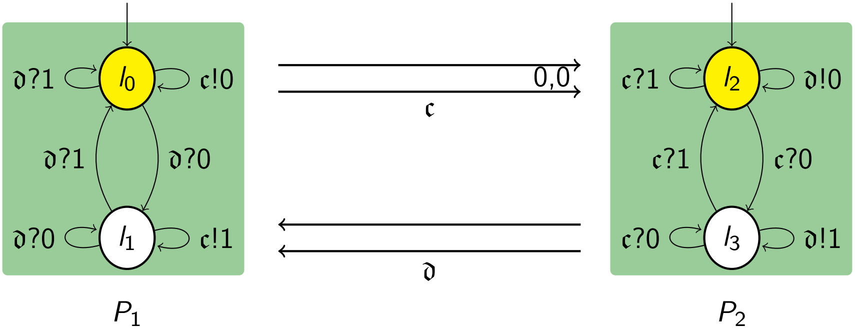

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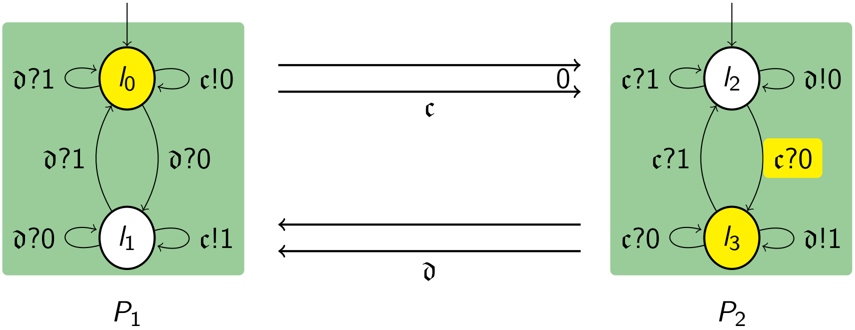

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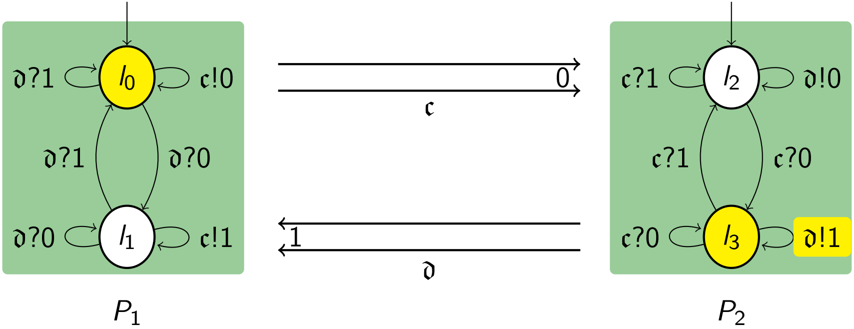

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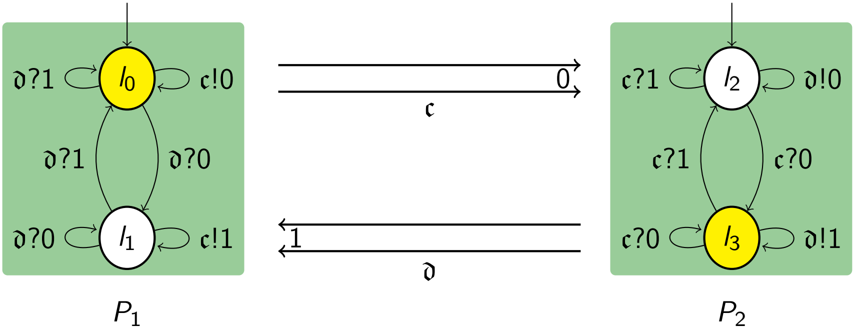

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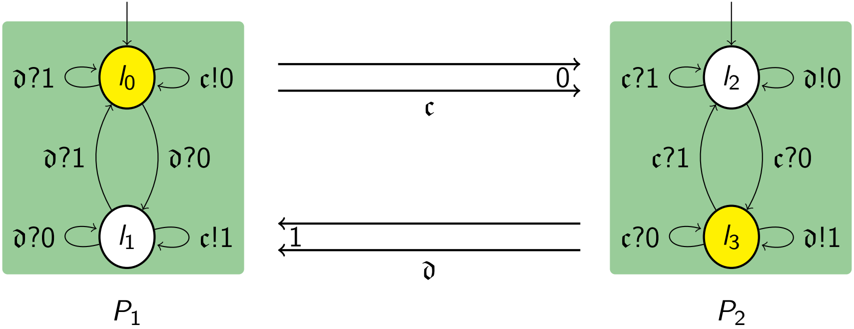
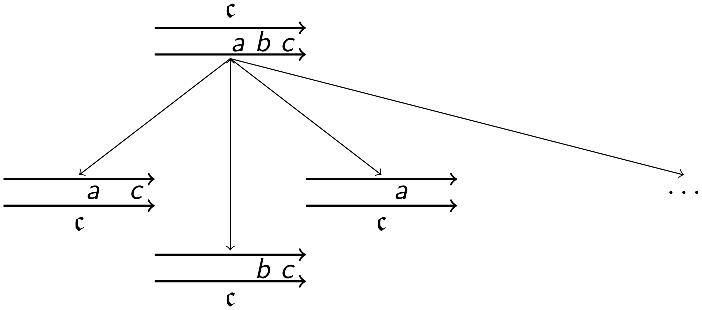


Communication:

- send a message m on $\mathfrak{c}$: $\mathfrak{c}!m$
- receive a message $\mathfrak{c}?m$ pop a message $m \in M$, **only if** m **was at the end of the queue**.

Lossiness assumption

Assumption: at every round, every message *may* disappear.



Effect of a transition (l, f, l') on state $s = l \cdot w$:

1. Change to location l';
2. Apply the channel operation operation f on w;
3. Drop from w to $w' \preceq w$: **subword ordering**.

Subword-ordering $\preceq$ is a well-quasi order [FS01]

Definition ([FS01])

$(S, \preceq)$ is a well-quasi-order (WQO) if:

$$\forall (s_i)_{i \in \mathbb{N}} \in S^{\mathbb{N}}, \exists i < j : s_i \preceq s_j$$

$(S, \to, \preceq)$ is a well-structured transition system:

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Theorem

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Sketch.

- Given $X \subseteq S$, the set $\mathrm{Pre}(X) = \{s \mid s \rightarrow X\} \subseteq S$ is **upward-closed** hence **regular**;
- Moreover, if X is a **regular** set, then $\mathrm{Pre}(X)$ can be computed.
- The computation $\cup_{i \geq n}\mathrm{Pre}^i(\{l_f\})$ converges in finite time.

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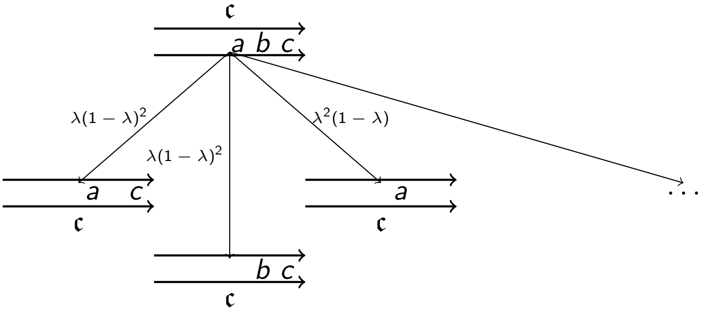
The result is written $[\![E(\Diamond R)]\!]$ and one just needs to check membership of l_0.

Lossiness in the probabilistic case

Stochastic case: the semantics is a Markov chain (S, Pr).

Local lossiness assumption: at every step, there is a positive probability $\lambda \in (0, 1)$, that a letter in the channel is dropped. Every message drop event is **independent** from the others.



Concurrent Games + Lossy Channel Systems

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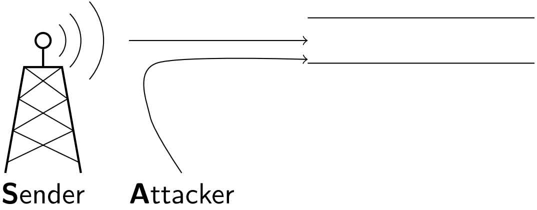
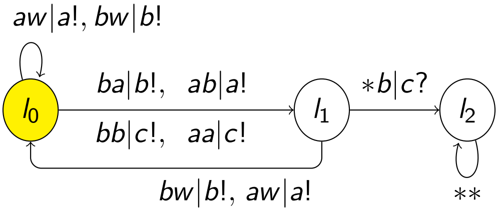
Infinite State Games

Example





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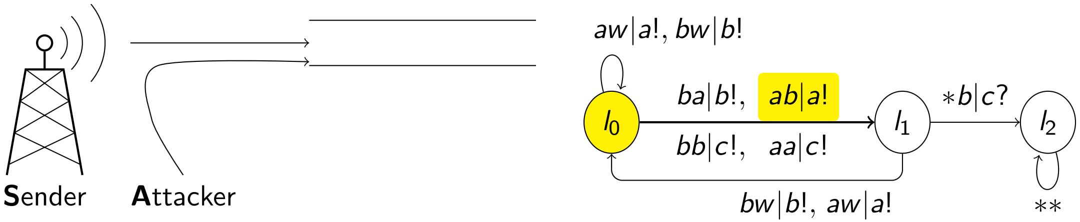

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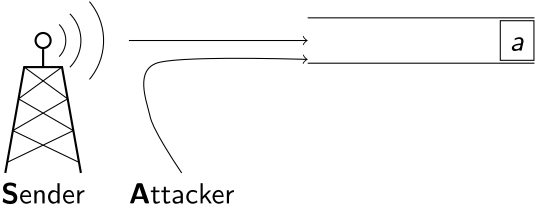

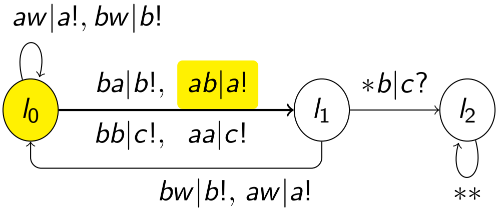

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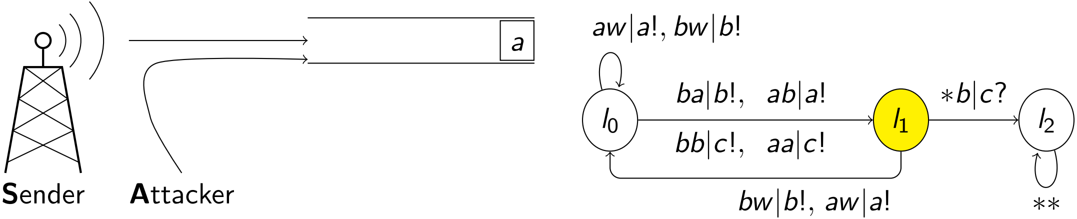

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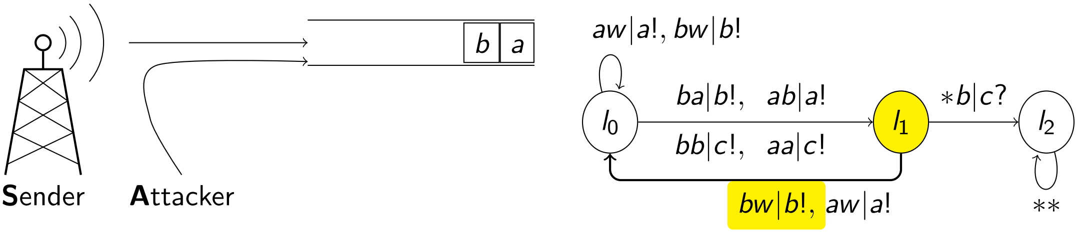

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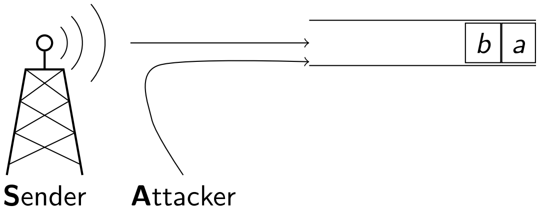

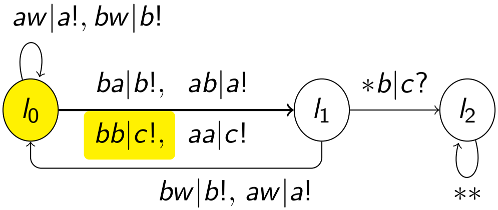

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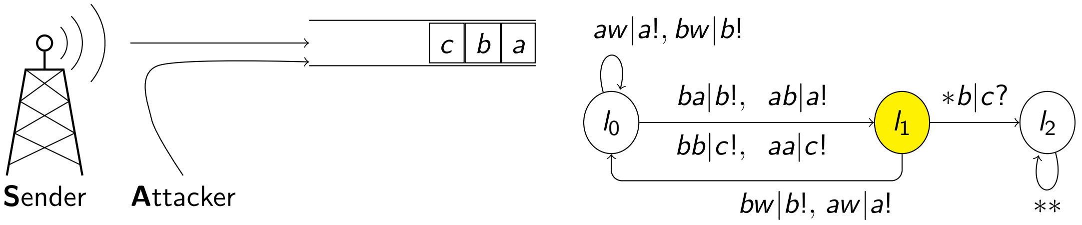

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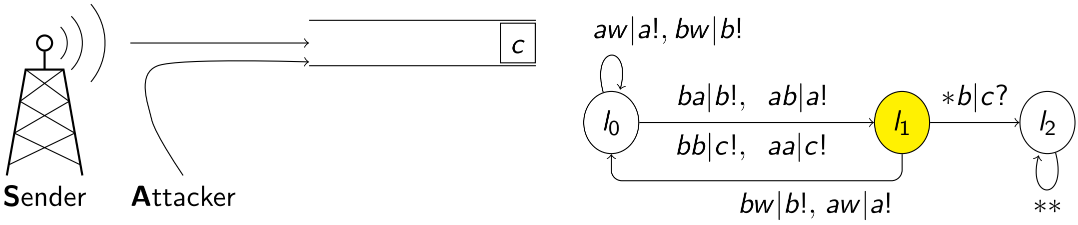

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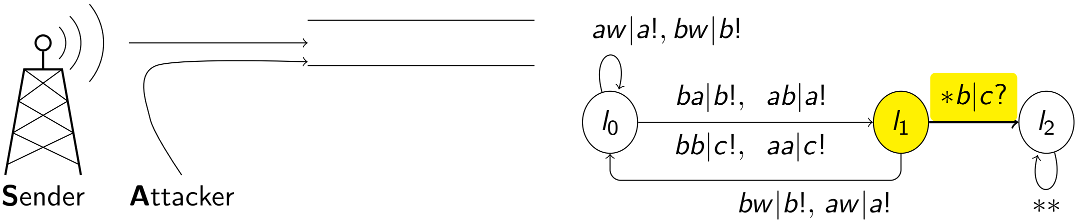

Zero-sum case

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- **Sure**/**Almost sure** *Safety:* $[\![\mathrm{Sure}(\Box R)]\!]$;
- **Positive P.** *Safety:* $[\![\mathrm{NZ}(\Diamond R)]\!]$;

Strategy Classes, Updated

A strategy for player i is: $\sigma_i : (L \cdot M^*)^+ \to \mathrm{Dist}(\mathrm{Act})$

- **D**etermistic: only one action with probability 1;

$$\forall h \in (L \cdot M^*)^+, \exists \alpha : \sigma_i(h)[\alpha] = 1$$

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P strategies **may** not be finitely represented. **PFM** are finitely represented, **Counting** too.

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NB: no hope for anything more:

-  Mayr [May03] proved that $[\![\mathrm{E}(\Box R)]\!]$ cannot be computed.
-  Bertrand and al [BBS07] proves that $\mathrm{NZ}(\Box\Diamond R)$ (Büchi) and $\mathrm{AS}(\Box\Diamond R_1 \wedge \Diamond\Box R_2)$ cases are undecidable.

Conclusion and future work

- Infinite state game with stochastic and concurrent behaviours;
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Future work:

- Restrictions on the strategy classes (FM only?);
- Support more combination of objectives to deal with Nash Equilibria;
- Partial observation (Signals).

Thank you for your attention

And have a nice trip back home :)

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