Regular Separability and Non-Determinizability of WSTS

Authors: Eren Keskin, Roland Meyer

Regular Separability

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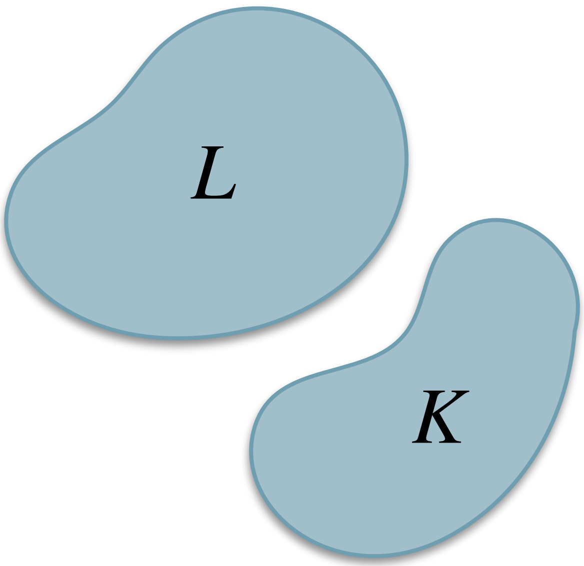

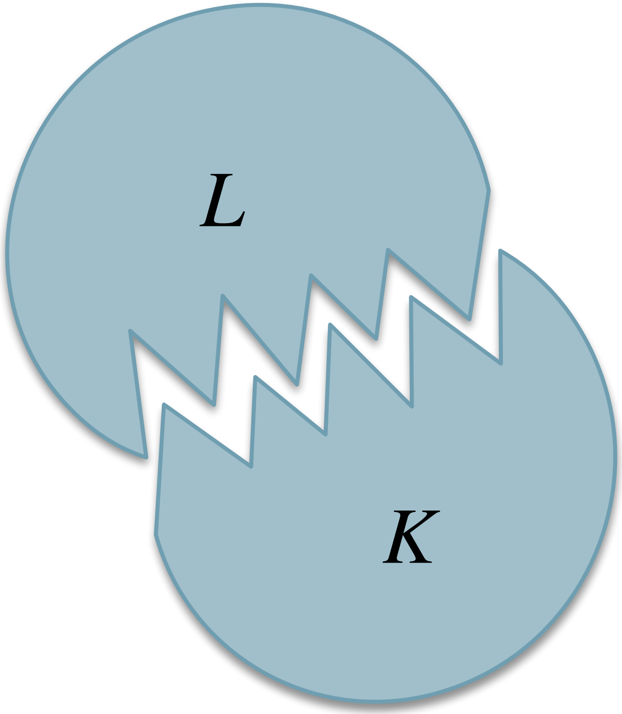

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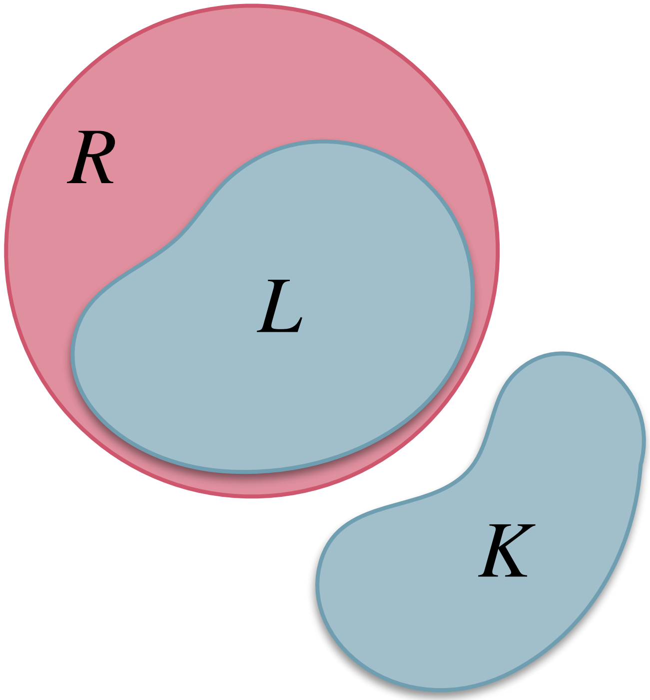

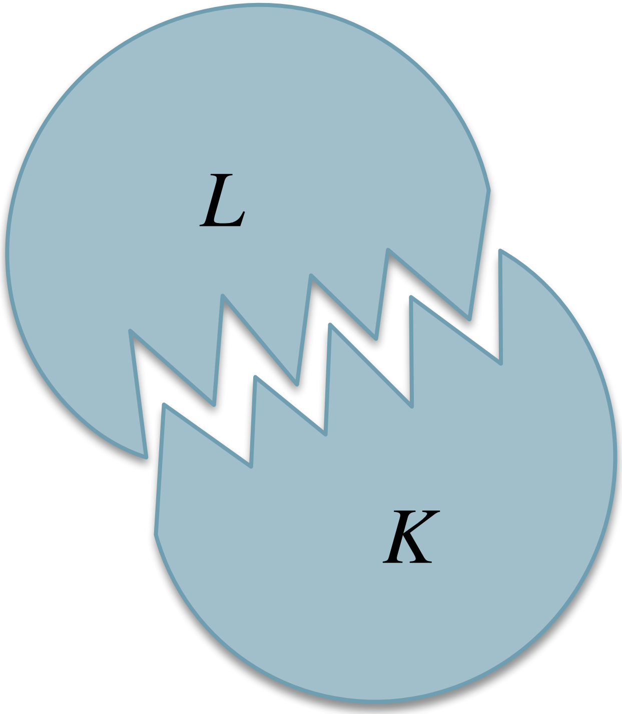

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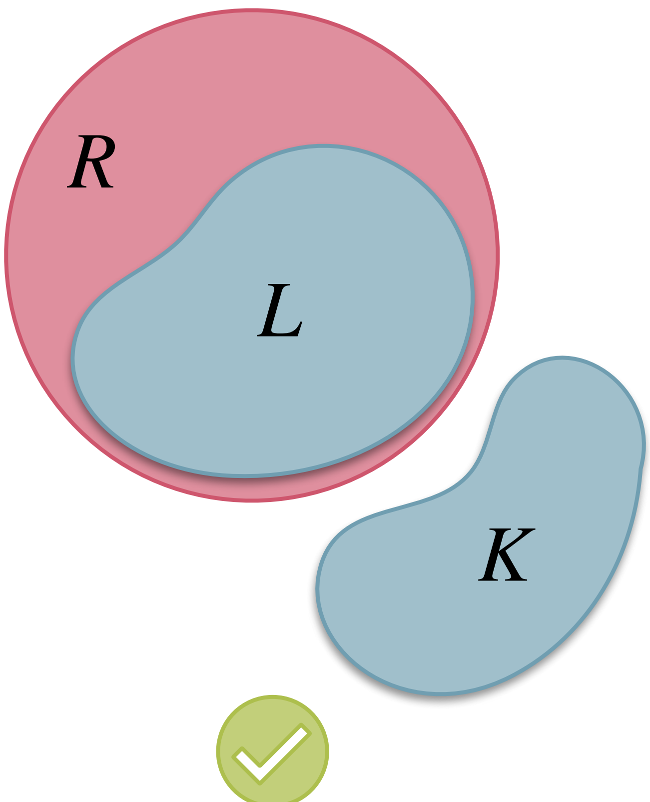
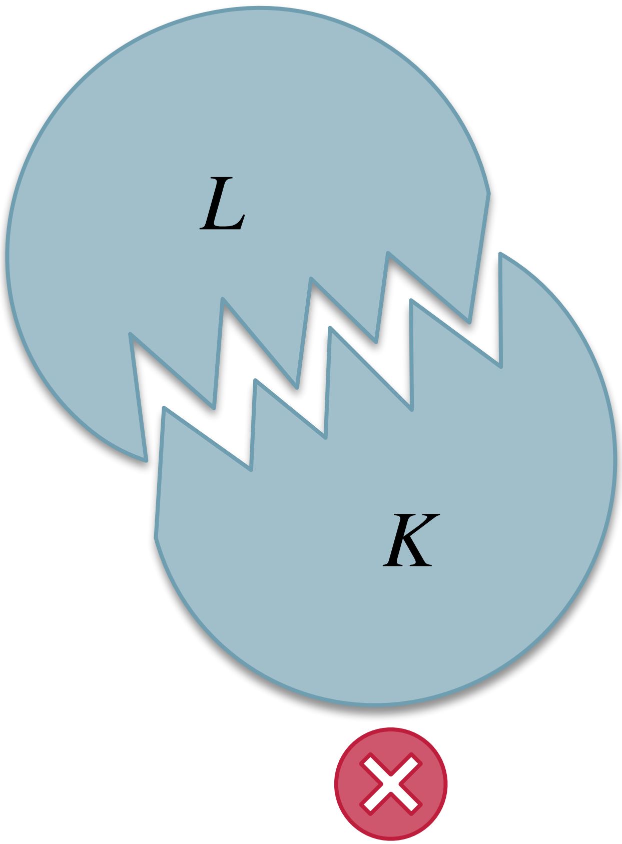
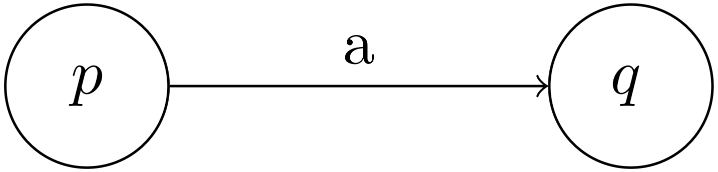




ULTS and WSTS

ULTS labeled transition system with monotony wrt. $\leq$



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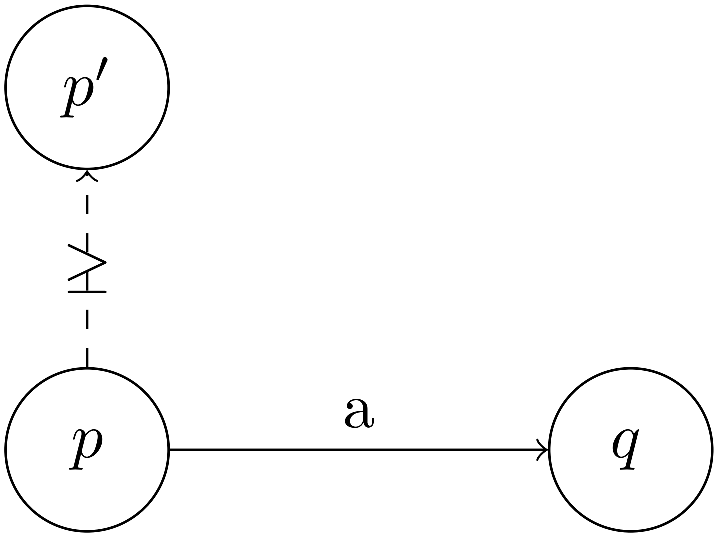

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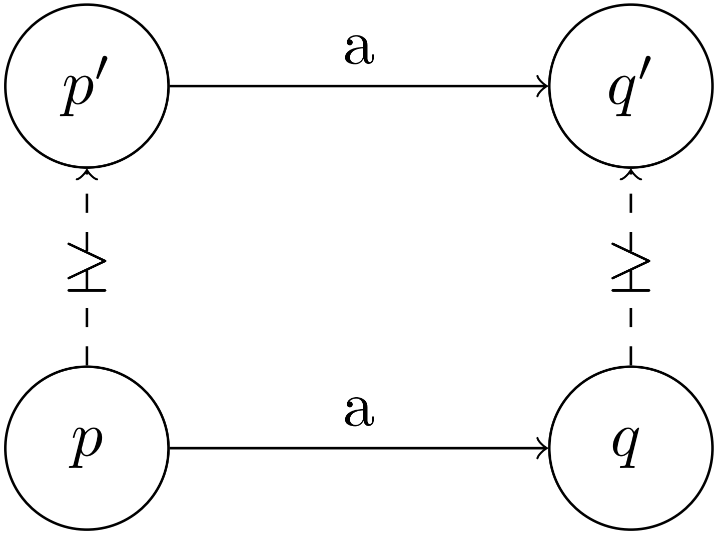

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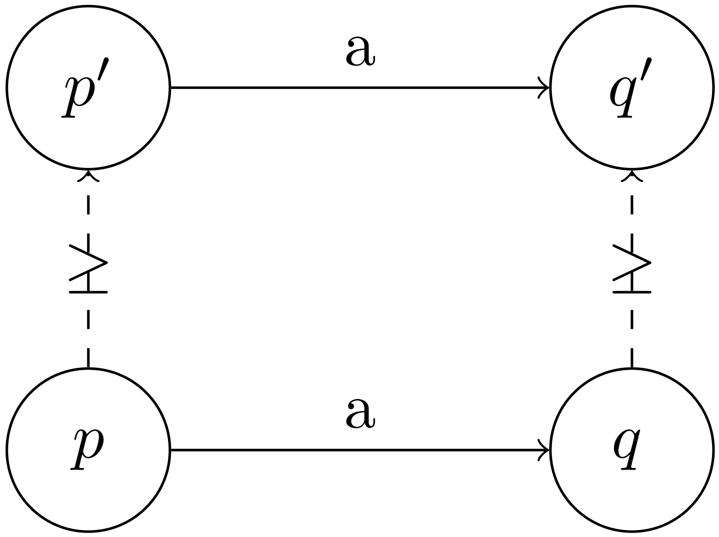

$(Q, \leq\ )$ WQO: WSTS

WQO

“Every infinite sequence has an increasing pair”

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$$q_0 \quad q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6 \quad q_7 \quad \cdots$$

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$$q_3 \leq q_7$$

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"Every infinite sequence has an increasing pair"

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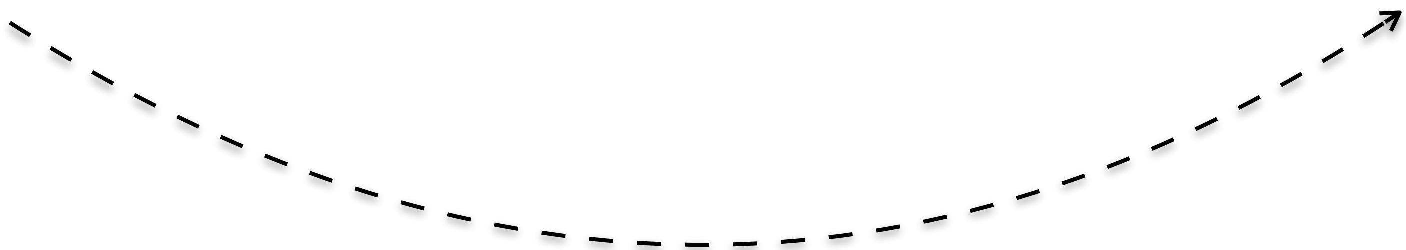

WSTS

Petri nets / VASS

Concurrent programs
under TSO

Lossy channel systems

Known Results [Czerwiński et al., CONCUR18]

Known Result I:

All pairs of disjoint WSTS languages are regularly separable **provided one is deterministic.**

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Known Result II:

Many WSTS can be determinized.

Determinizability

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Example: U VAS

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U State space: $(\mathbb{N}^k, \leq)$

U_{det} State space: $(\mathbb{D}(\mathbb{N}^k), \subseteq)$

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↑

WQO! (ω^2-WQO)

Our Contributions [CONCUR 23]

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New Result I:

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Known Result II:

Many WSTS can be determinized.

New Result II:

Some WSTS can not be determinized.

Regular Separability

Technische Universität Braunschweig

Proof Technique in [Czerwiński et al., CONCUR 18]

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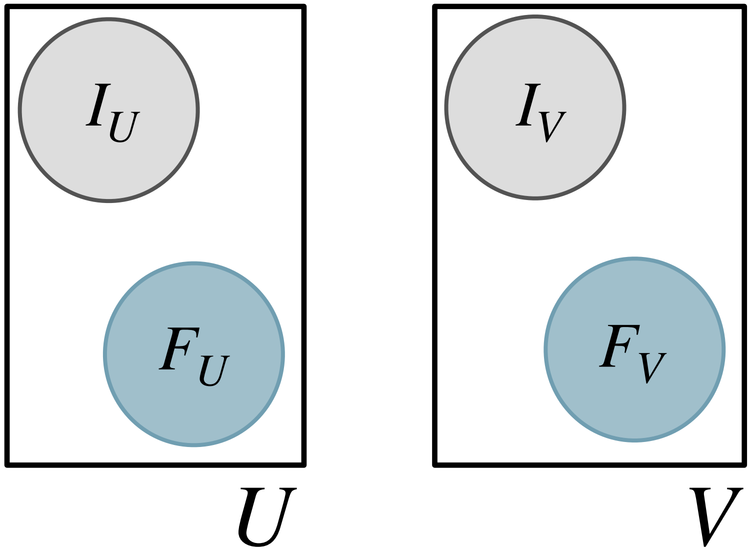

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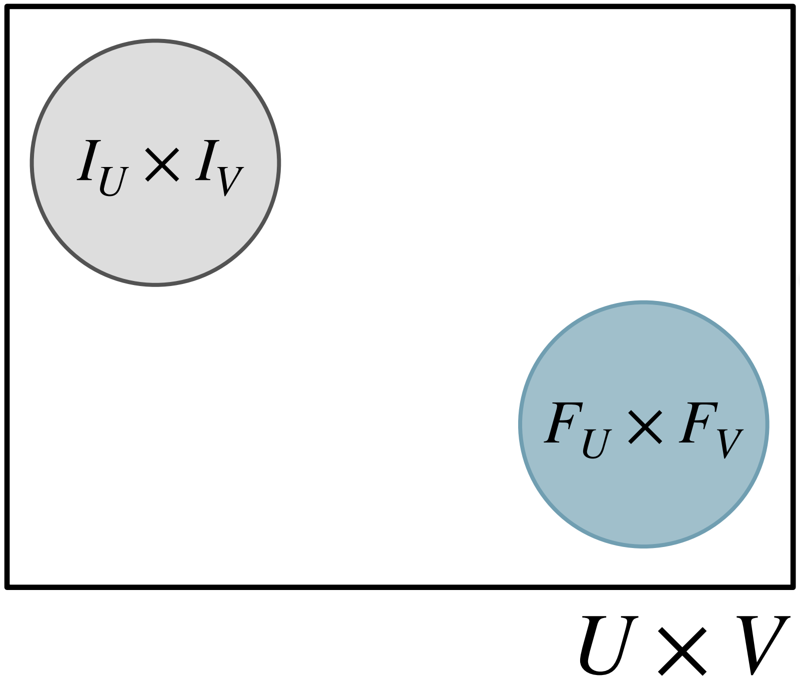

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Product: $U \times V$

$$L(U \times V) = \emptyset$$

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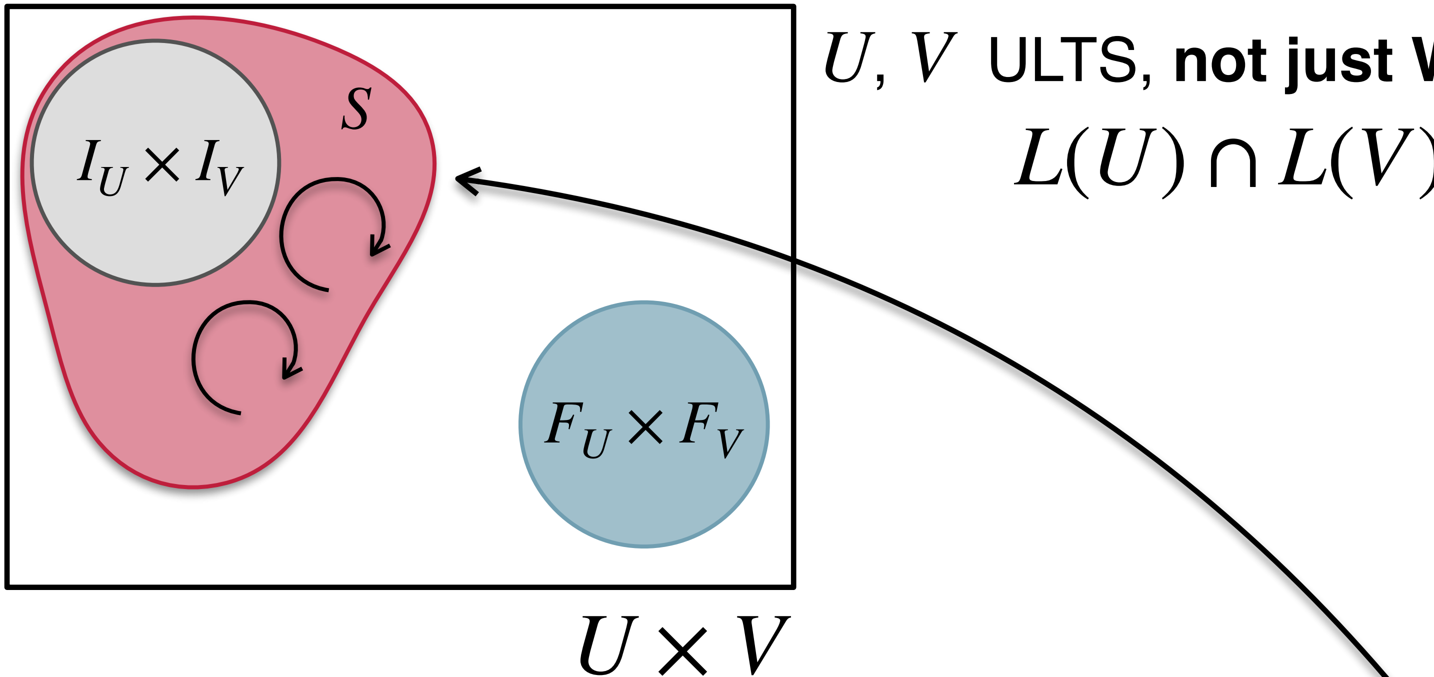

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$$L(U \times V) = \varnothing \implies$$

Inductive invariant S

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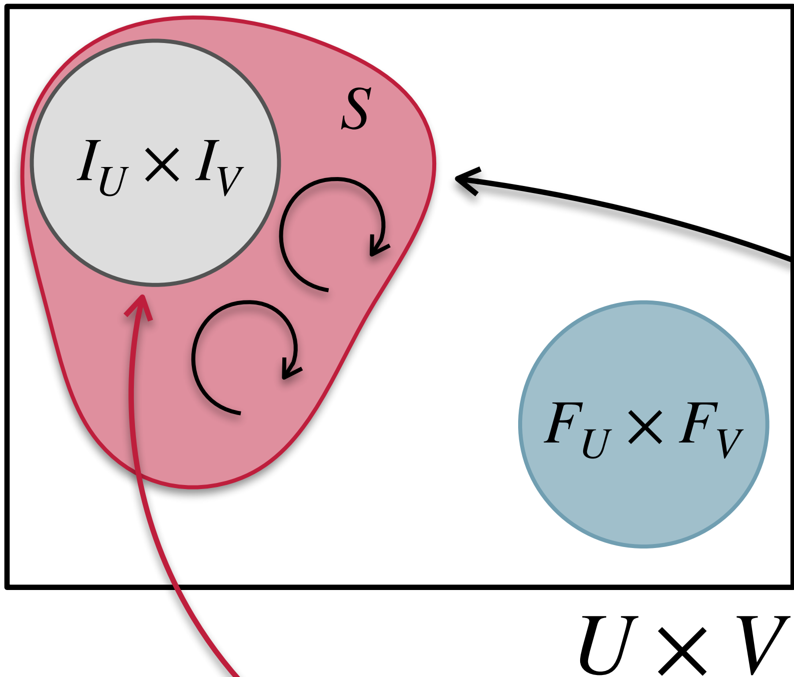

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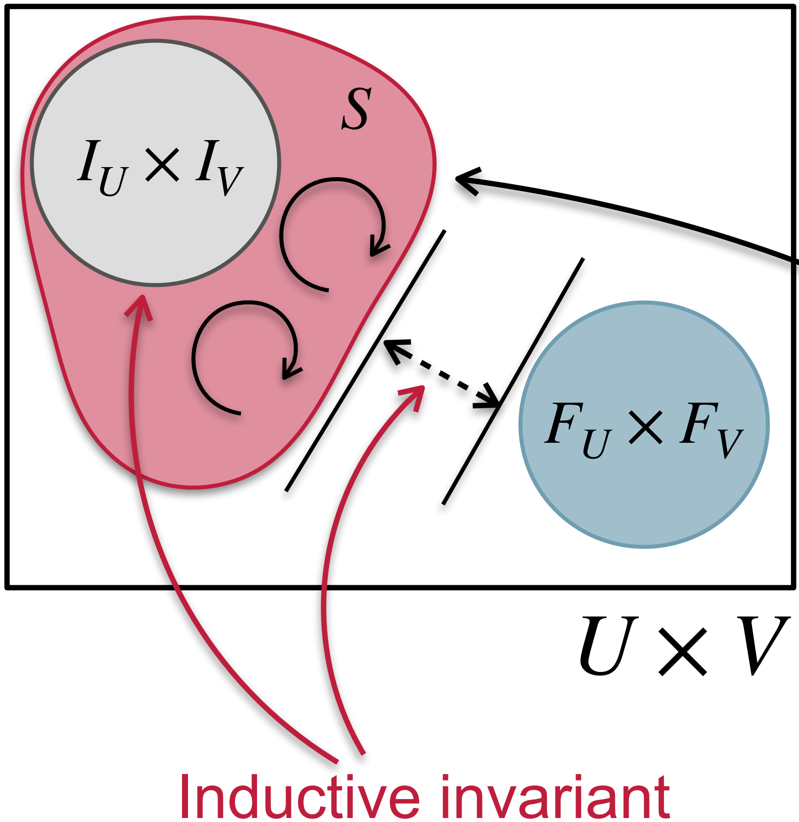

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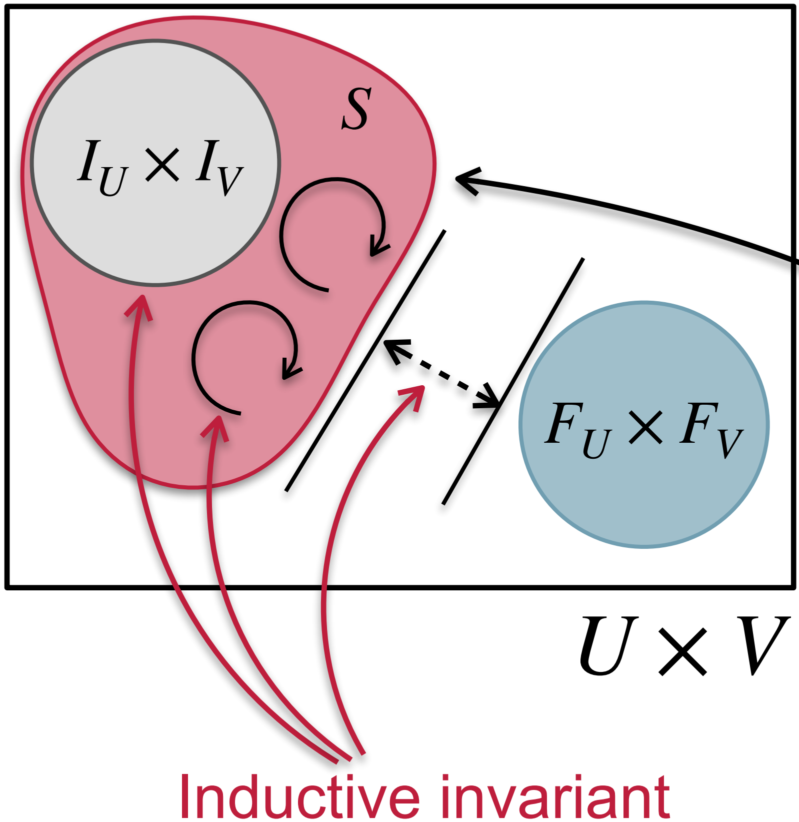

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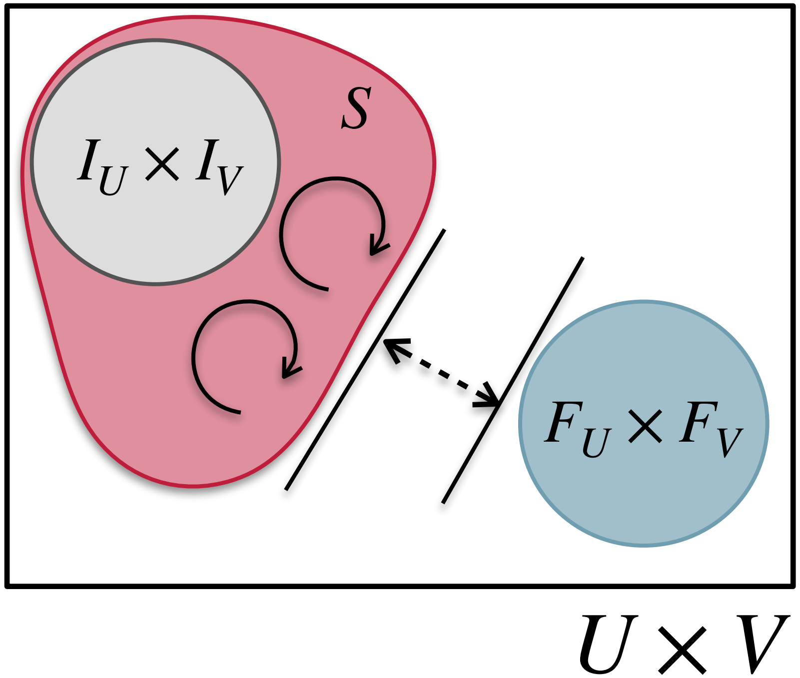


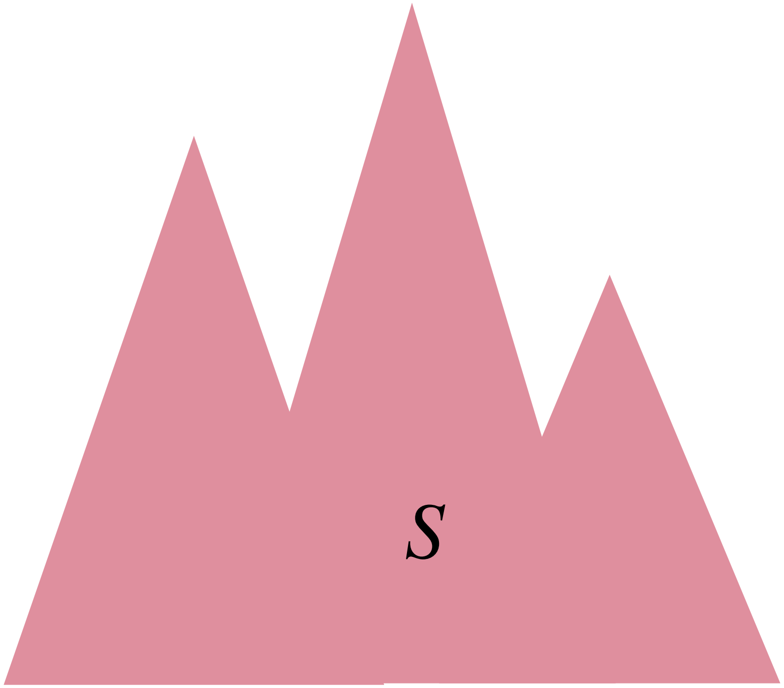
U, V ULTS, **not just WSTS!**, one det.

$$L(U) \cap L(V) = \emptyset$$

Finitely rep. inductive invariant in $U \times V \Longrightarrow L(U), L(V)$ **reg. sep**

Proof Technique in [Czerwiński et al., CONCUR 18]

Finite representation of S:



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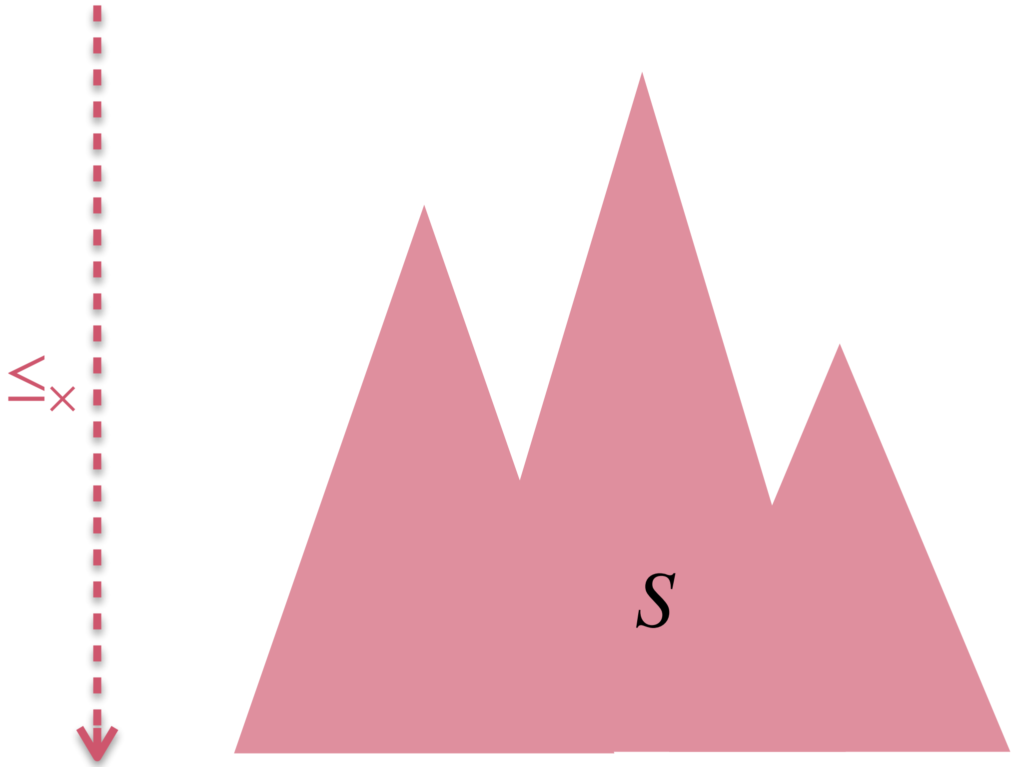

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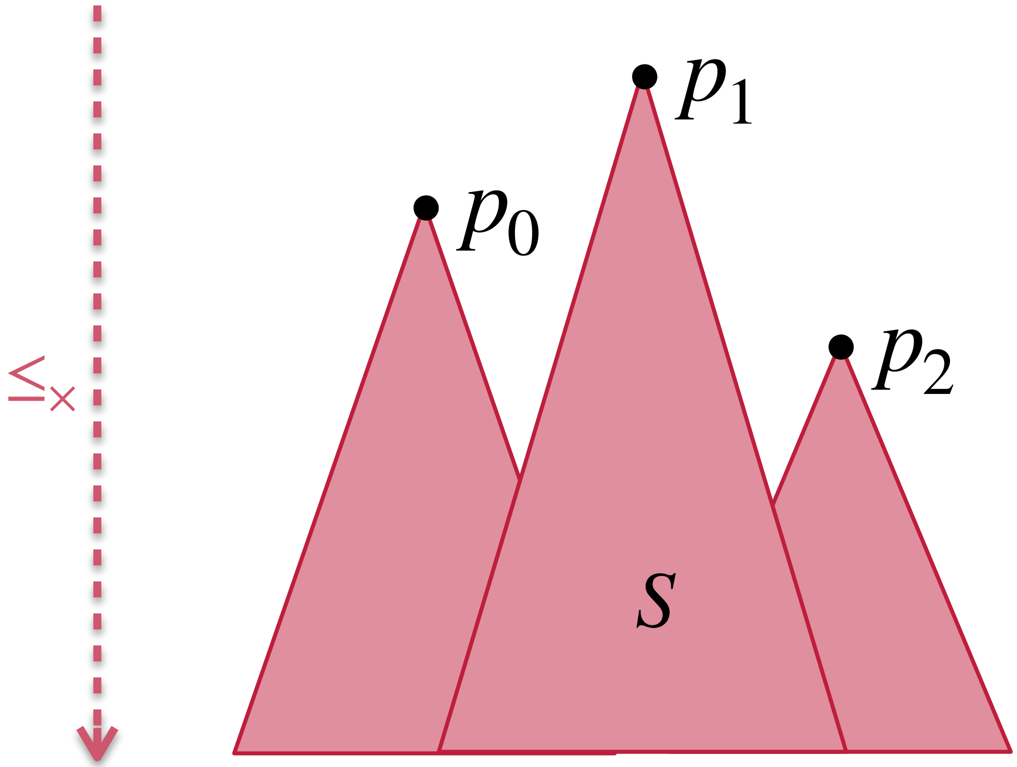

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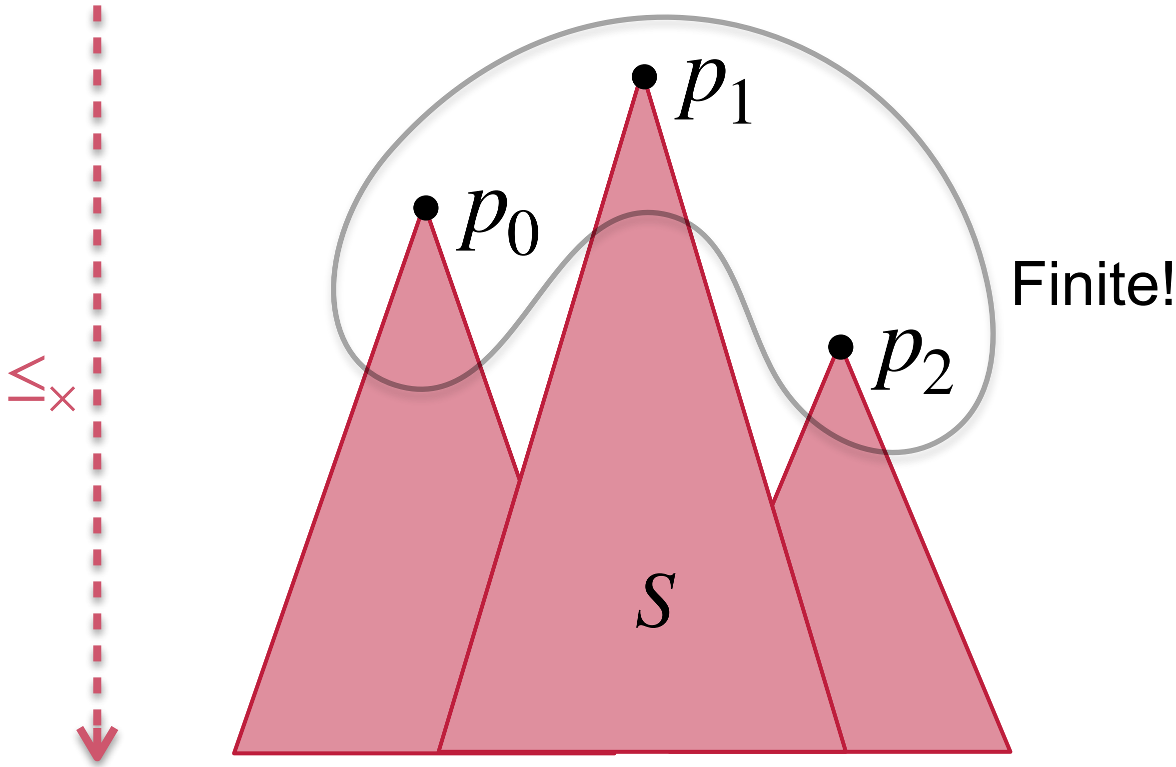

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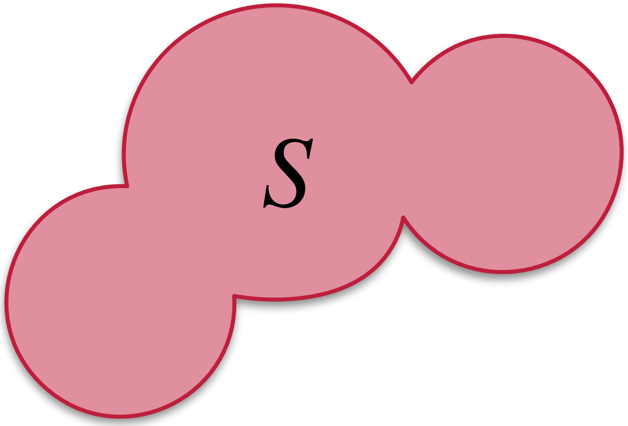

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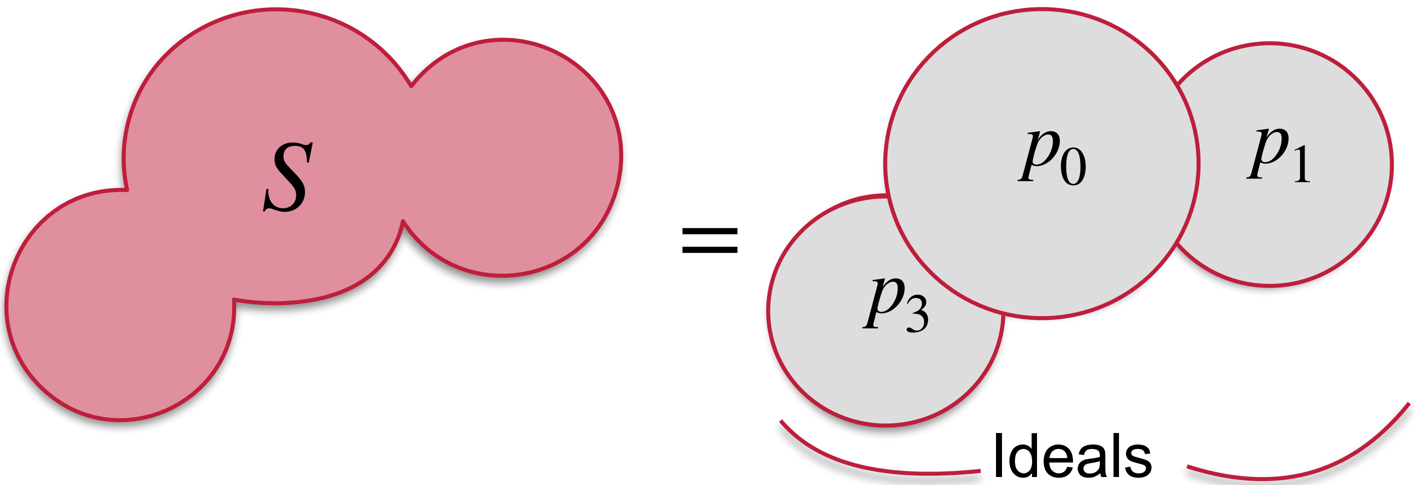

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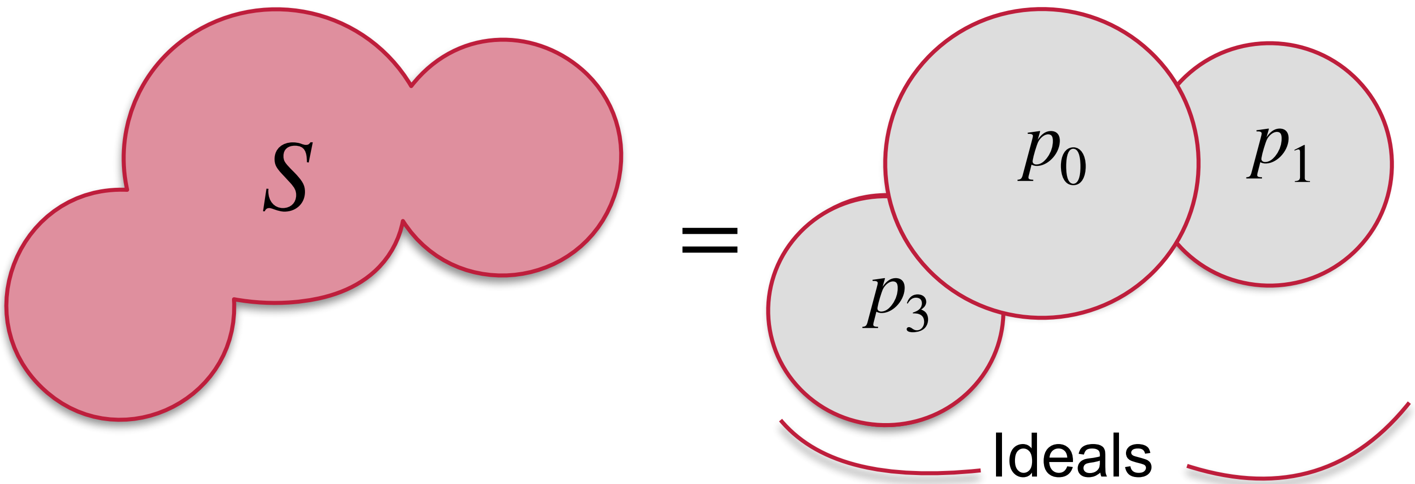

BUT: Determinization breaks WQO

Key Problem

Find **finitely rep.** inductive invariants *without* using ideals.

Our Approach

U V

Get a pair of WSTS

$L(U) \cap L(V) = \varnothing$

Our Approach

U_{det} V_{det} Determinize!

Our Approach

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Determinize!

No longer WSTS, accept it!

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$$U_{det} \times V_{det}$$

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Inductive Invariant S

Determinize!

No longer WSTS, accept it!

Our Approach

$$U_{det} \times V_{det}$$

$\uparrow$

S

Determinize!

No longer WSTS, accept it!

Exploit the remaining properties.

Our Approach

Which properties?

Our Approach

Which properties?

Key Insight [Rado, 54]

All sequences of downward closed subsets have ***converging*** *subsequences.*

Convergence

$q_0, q_1, \ldots$ **converges if** $\bigsqcup_{i \in \mathbb{N}} \bigsqcap_{j \geq i} q_j = \bigsqcup_{i \in \mathbb{N}} q_i$ ← The limit

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$q_0 = \{a\} \quad q_1 = \{b\} \quad q_2 = \{a, c\} \quad \ldots \quad q_n = \{b, c\} \quad q_{n+1} = \{b, a\} \quad \ldots$

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Lemma

$cl(S)$ represented by finitely many max. Elements

Proof: Zorn's Lemma

Non-Determinizability

Question:

$$L(\text{Deterministic WSTS}) = L(\text{All WSTS})?$$

Is non-determinism more expressive?

[Czerwiński et al., CONCUR 18]

ω^2-WSTS ✓

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Finitely branching WSTS ✓

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We show

Infinitely branching, non ω^2-WSTS ✗

What can det. WSTS do?

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Problem: Expressiveness of det. WSTS

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<u>Solution:</u> Myhill-Nerode-esque approach

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$. x_1 \notin L$

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$. x_3 \in L$

$. x_4 \notin L$

$. x_5 \in L$

$\vdots$

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$$\Longrightarrow$$

$$v.x_0 \in L,\ v.x_2 \in L,\ v.x_3 \in L,\ v.x_5 \in L,\ \ldots$$

What can det. WSTS do?

<u>Problem:</u> Expressiveness of det. WSTS

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Lemma

L accepted by deterministic WSTS **iff** $(\Sigma^*, \leq_L)$ WQO.

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Wanted: Language with infinite anti-chain

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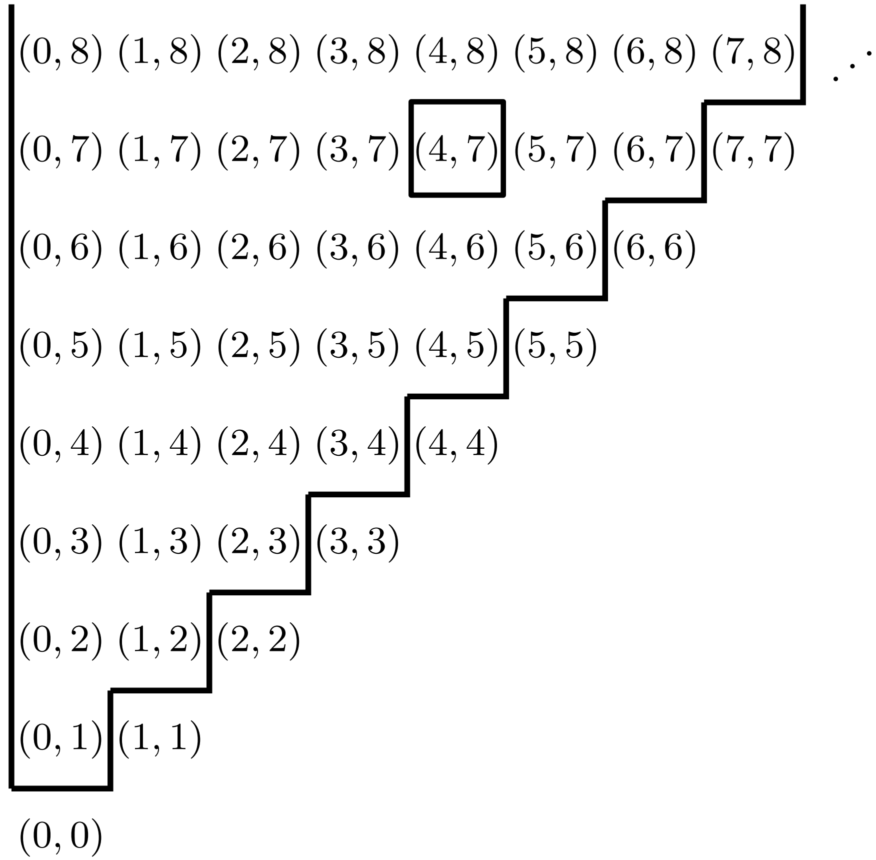
=WQO embeds the Rado WQO!

The Rado WQO



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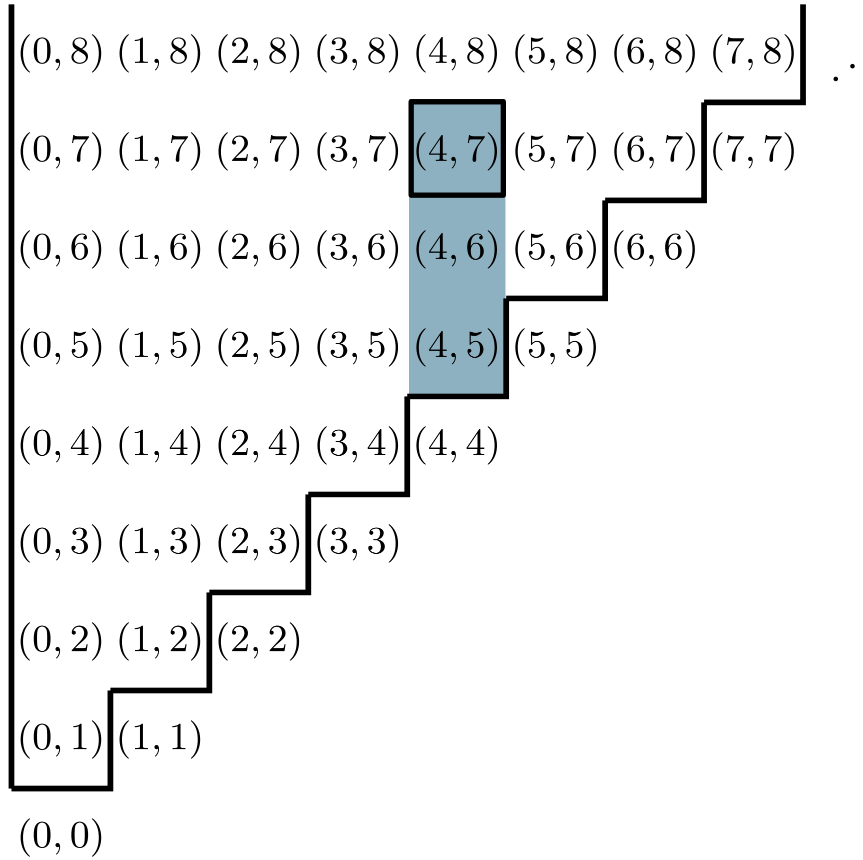

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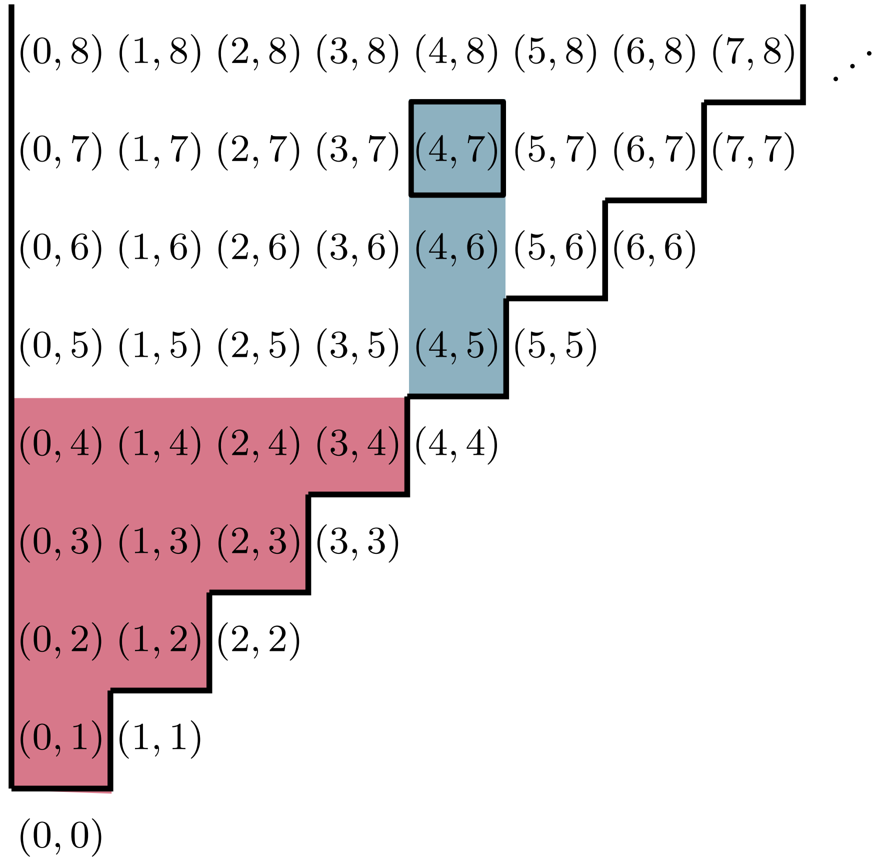

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Downward Closed Subsets of the Rado WQO

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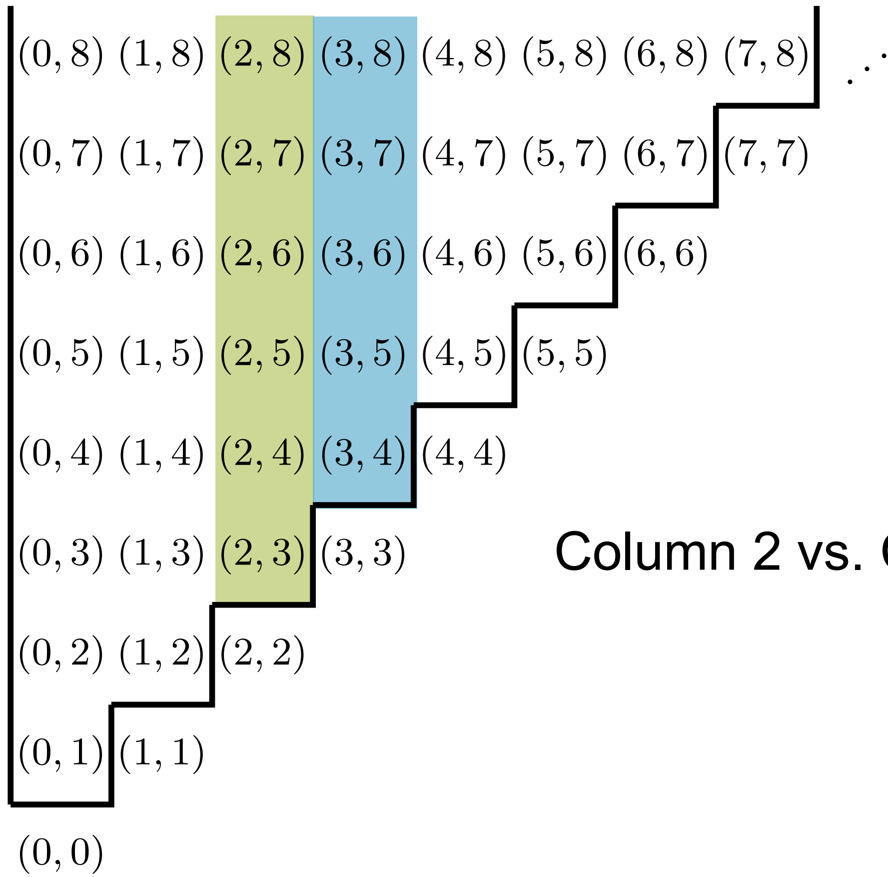

Column 2 vs. Column 3

Downward Closed Subsets of the Rado WQO

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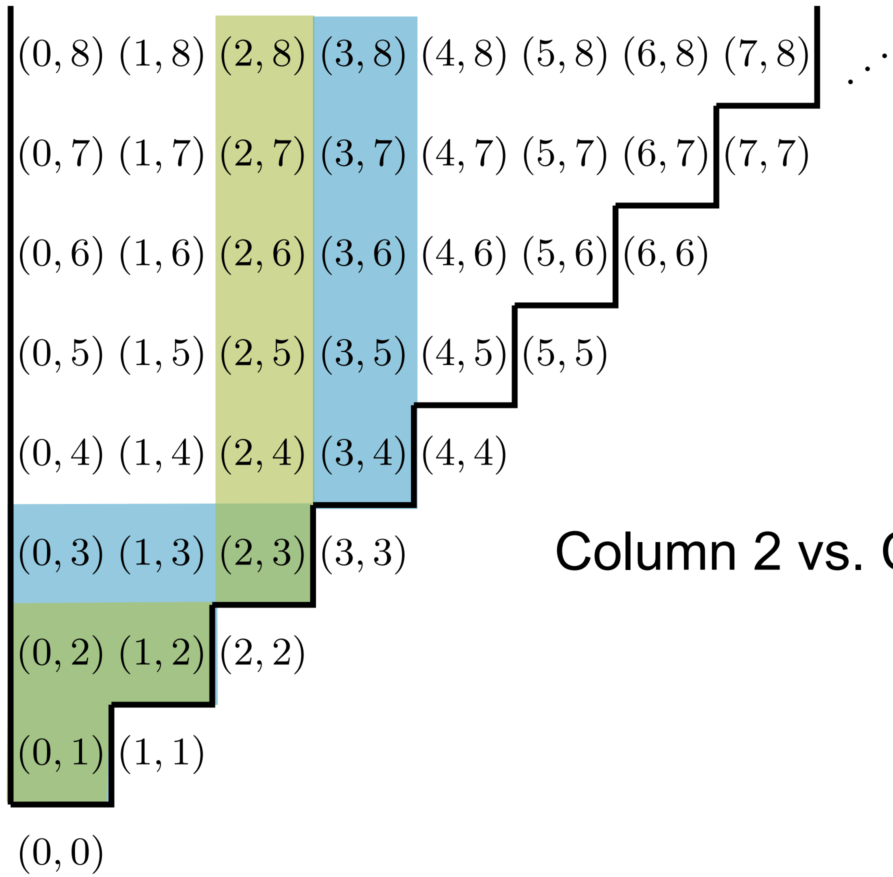

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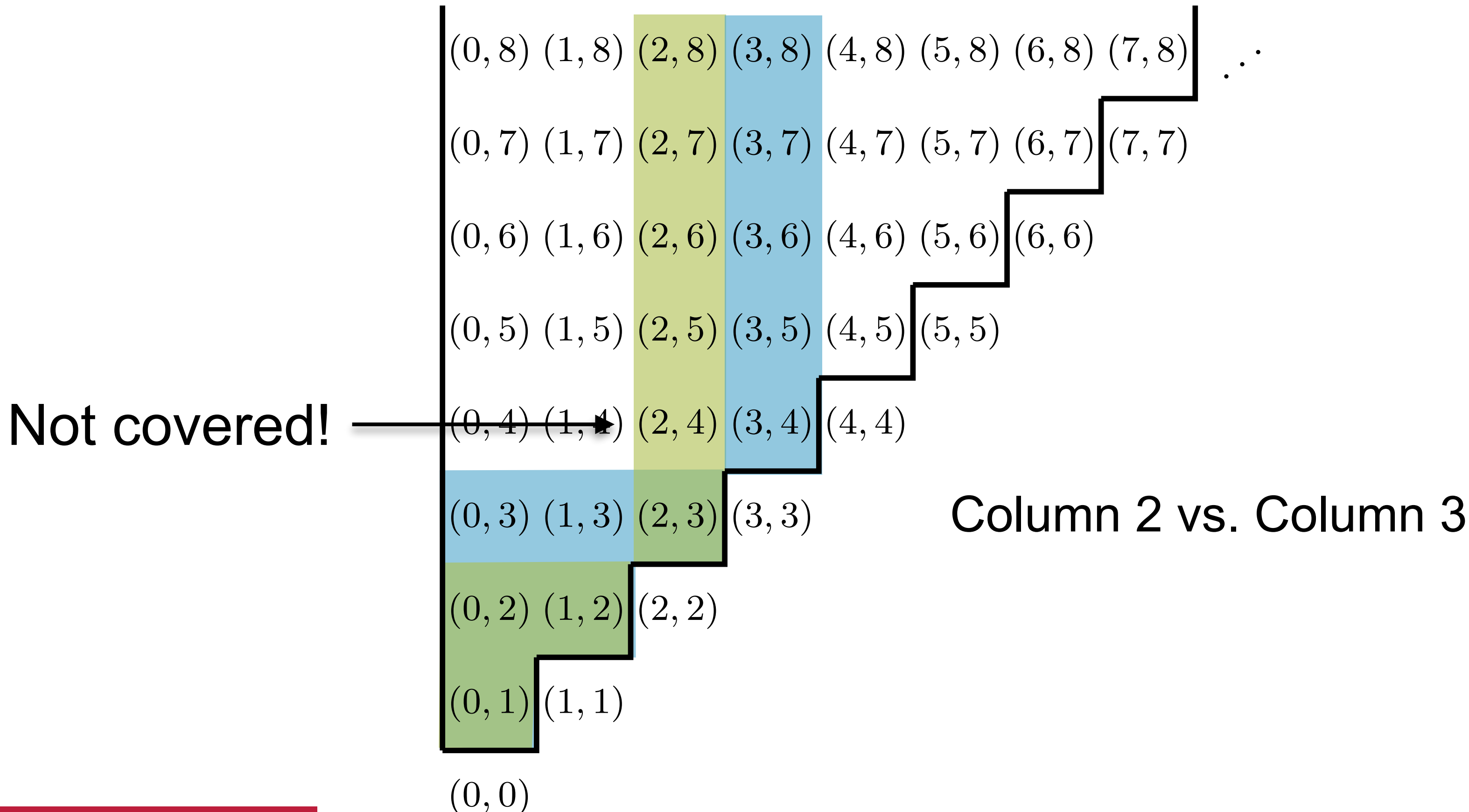

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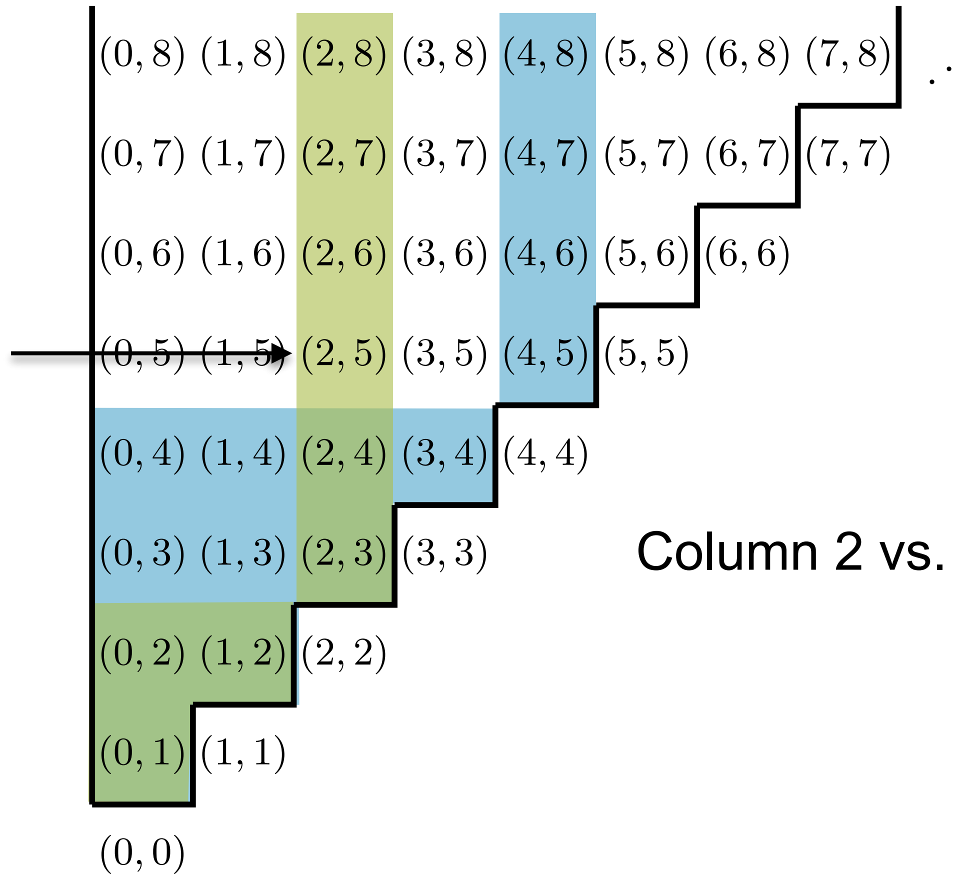

Column 2 vs. Column 4

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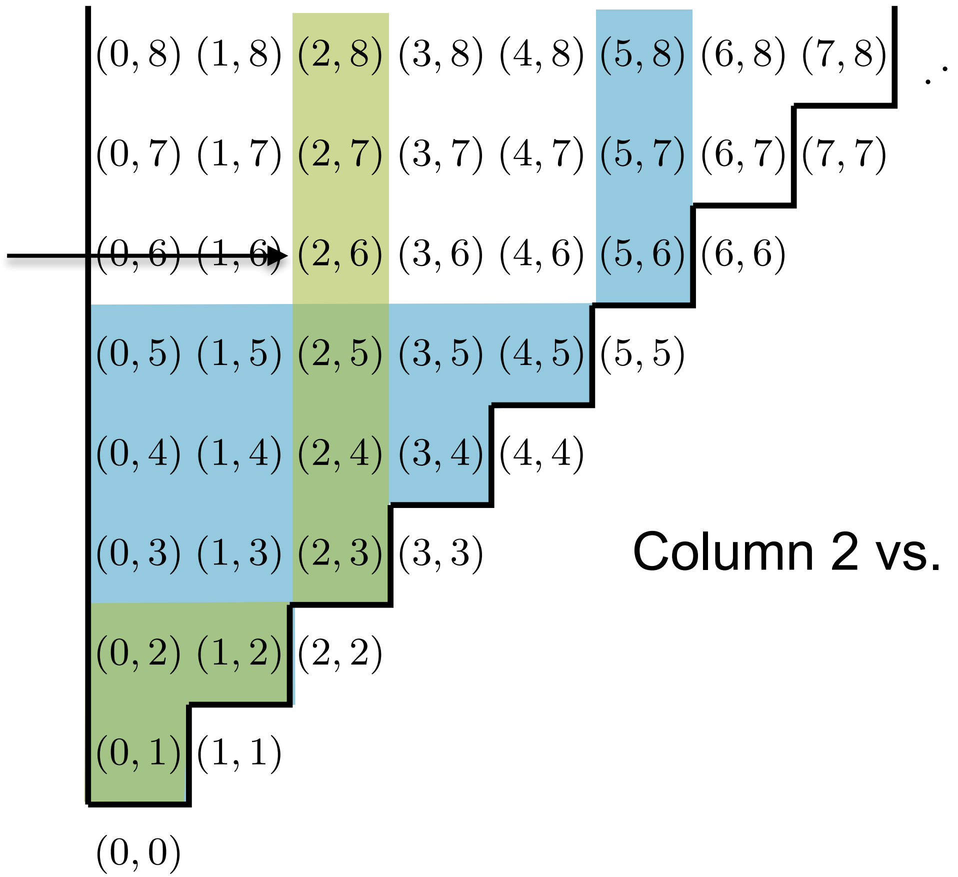

Column 2 vs. Column 5

The Witness Language

Wanted: Language with infinite anti-chain

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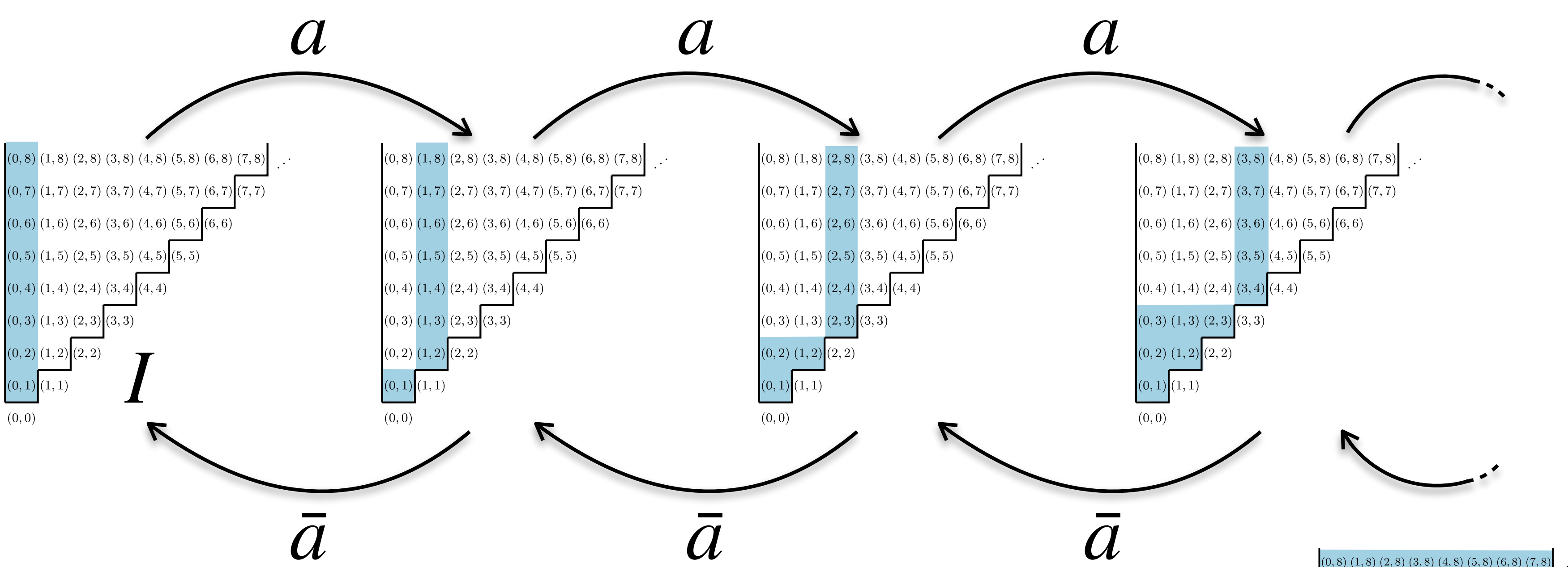

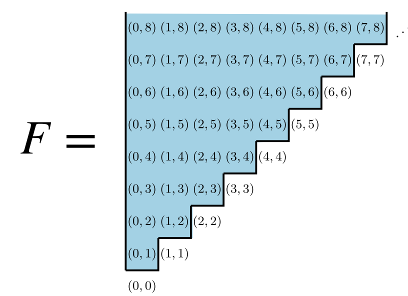

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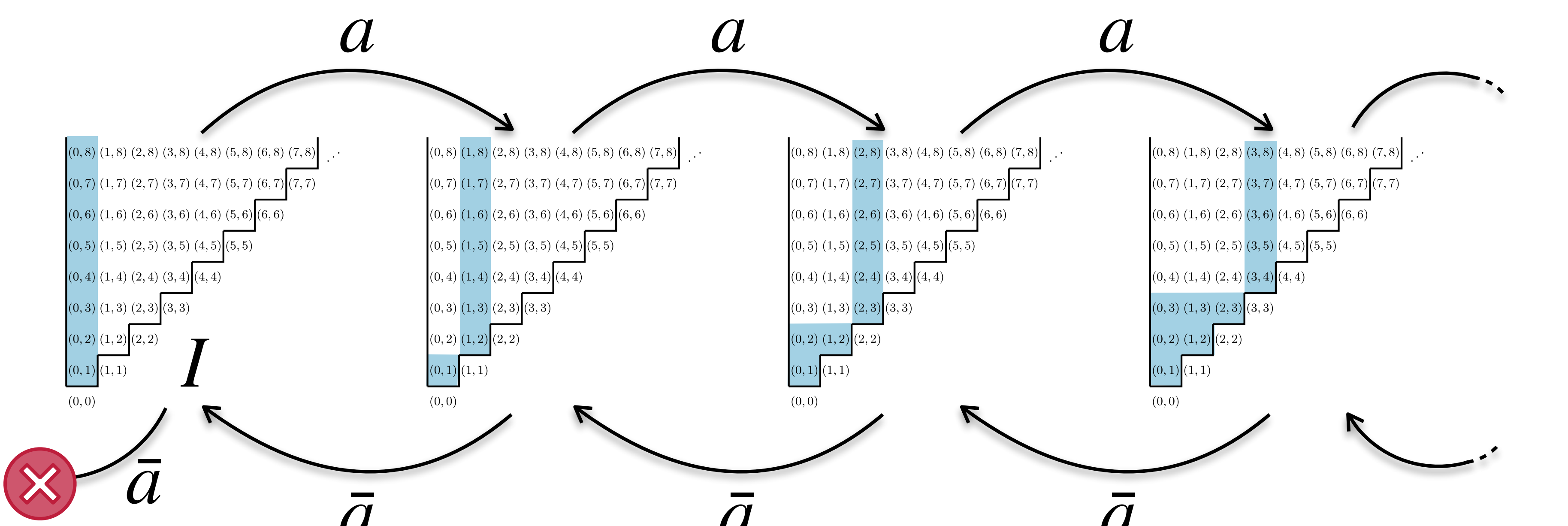
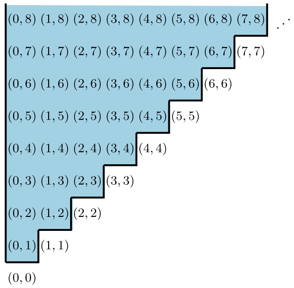


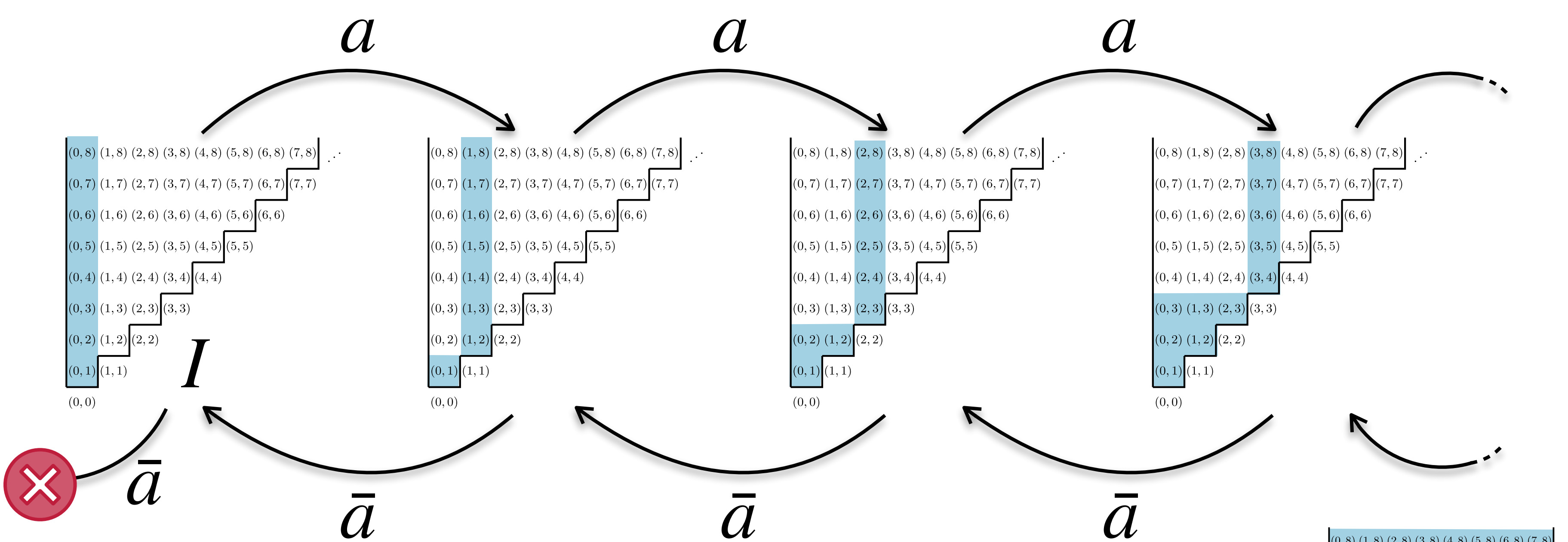
$$a^{n+k} \not\leq_T a^n$$

$F =$



The Witness Language

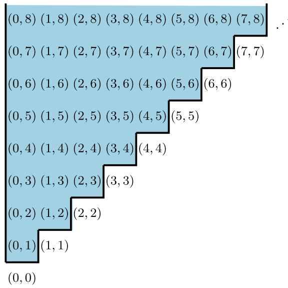
Wanted: Language with infinite anti-chain



Column 0: Stable under $zero$

Column n: Fails after $zero^n$

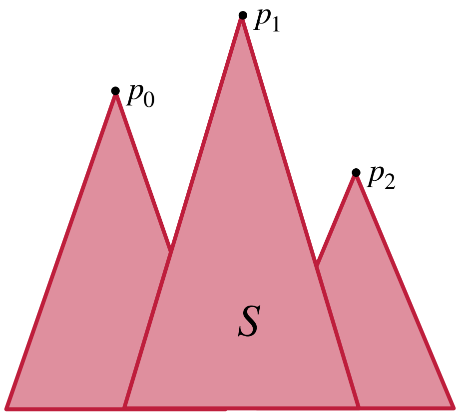
$a^n \not\leq_T a^{n+k}$

$F =$ 

Conclusion

Conclusion

We have shown: All disjoint WSTS pairs are regularly separable.



$$\bigsqcup_{i \in \mathbb{N}} \bigsqcap_{j \geq i} q_j = \bigsqcup_{i \in \mathbb{N}} q_i$$

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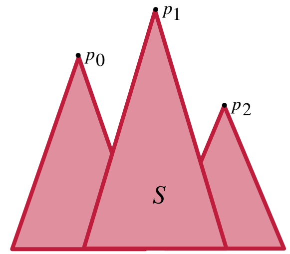
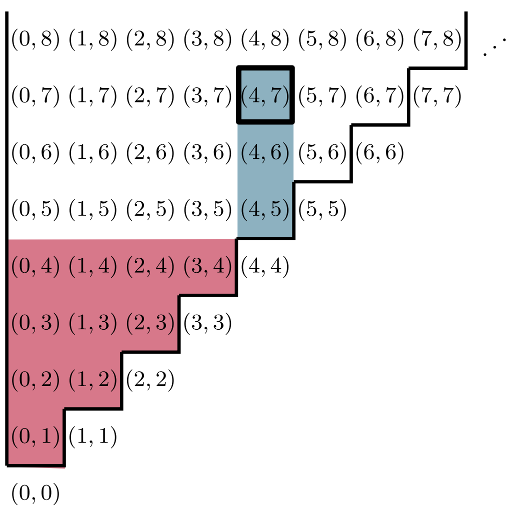
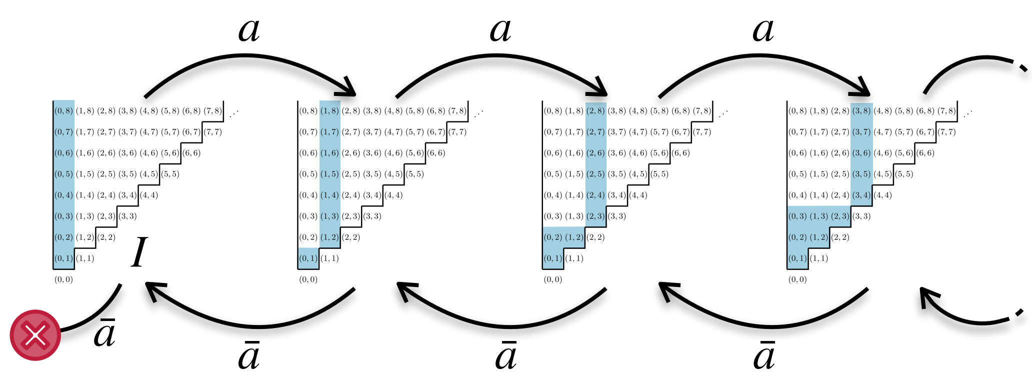


$$\bigsqcup_{i\in\mathbb{N}} \bigsqcap_{j\geq i} q_j = \bigsqcup_{i\in\mathbb{N}} q_i$$

We have shown: Non-det. is strictly more expressive.





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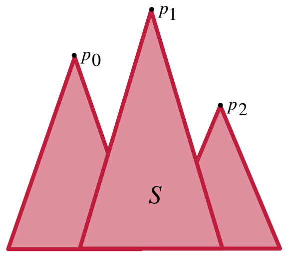

$$\bigsqcup_{i\in\mathbb{N}}\bigsqcap_{j\geq i} q_j = \bigsqcup_{i\in\mathbb{N}} q_i$$

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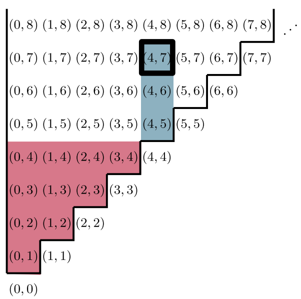
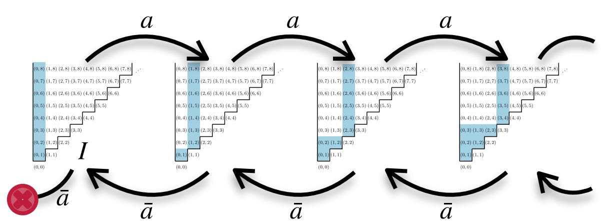
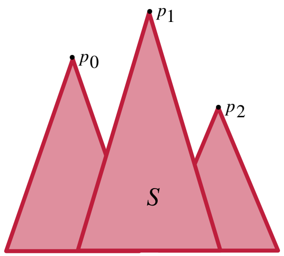




In the paper: All results also hold for downward WSTS

Conclusion

We have shown: All disjoint WSTS pairs are regularly separable.



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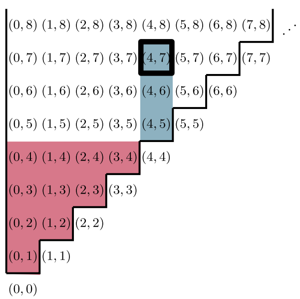

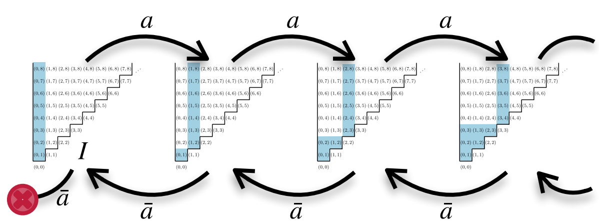

In the paper: All results also hold for downward WSTS

… and many more relationships between the classes!

Appendix

Closed Form of a Fragment of the witness language T

$$T \cap a^*\bar{a}^*zero^* = \{a^n\bar{a}^n zero^k \mid n, k \in \mathbb{N}\} \cup \{a^n\bar{a}^k zero^l \mid n, k, l \in \mathbb{N}, n - k > l\}$$